

GENERALIZED NETS WITH INHIBITOR ARCS

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Abstract

A new extension of the class of the ordinary Generalized Nets (GNs) is defined – GNs with Inhibitor Arcs (GNIA). In a GNIA, some of the arcs can inhibit the tokens that are transferred through them. Algorithm for transfer of tokens in GNIA and algorithm for GNIA functioning are described. It is proved that the class of all GNIAs is a conservative extension of the class of the ordinary GNs. The new extension can be used in the modelling of information service networks, such as overall telecommunication systems.

Key words: generalized nets, generalized nets extensions, generalized nets with inhibitor arcs

2020 Mathematics Subject Classification: 65Q85

Introduction. The Generalized Nets (GNs) were introduced in 1982 as extensions of Petri nets (see, e.g., [1–5]). In recent years, they have been extended in different directions. For each of these extensions it is proved that it is a conservative one, i.e., the functioning and the result of work of any net from the extensions can be described by a standard GN.

In the present paper, we present a formal definition of a new GN extension – Generalized Nets with Inhibitor Arcs (GNIA). It is proved that the new GN extension is conservative. Some of its possible applications are discussed.

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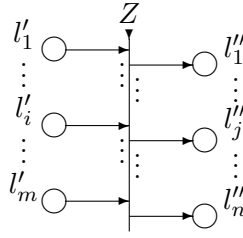


Fig. 1. Graphical representation of a transition of a GNIA

Generalized nets with inhibitor arcs. Every transition of a GNIA has the same components as the transitions of the ordinary GN but the Index Matrix (IM; for index matrices see, e.g., [6]) of the transition's condition has a different form. In GNIA, the elements of the IM of the transition's condition are ordered pairs of predicates. Formally, every transition of a GNIA is described by a seven-tuple (see Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r^*, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively). For the transition in Fig. 1 these are $L' = \{l'_1, l'_2, \dots, l'_m\}$ and $L'' = \{l''_1, l''_2, \dots, l''_n\}$.

(b) t_1 is the current time moment of the transition's firing.

(c) t_2 is the current value of the duration of its active state.

(d) r^* is the transition's *condition* determining which tokens will be transferred from the transition's input places to its output places. Parameter r^* has the form of an IM:

$$r^* = \begin{array}{c|ccc} & l''_1 & \dots & l''_j & \dots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & & & \\ l'_i & \langle r_{i,j}^1, r_{i,j}^2 \rangle & & & & \\ \vdots & (r_{i,j}^1, r_{i,j}^2 - \text{predicates}) & & & & \\ l'_m & (1 \leq i \leq m, 1 \leq j \leq n) & & & & \end{array} ,$$

where $r_{i,j}^1$ is the predicate which expresses the condition for transfer from the i -th input place to the j -th output place. When $r_{i,j}^1$ has truth-value "true", then a token from the i -th input place can be transferred to the j -th output place, otherwise this is impossible. Predicate $r_{i,j}^2$ determines whether the current token in the input place l'_i will be inhibited by the arc between the input place l'_i and the output place l''_j . When the arc between an input place l'_i and output place l''_j does not inhibit tokens, then $r_{i,j}^2 = \text{"false"}$. This component of the GNIA transitions is different from its analogue in the ordinary GNs.

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \dots l''_j \dots l''_n \\ \hline l'_1 & \\ \vdots & \\ l'_i & (m_{i,j} \geq 0 - \text{natural number or } \infty) \\ \vdots & \\ l'_m & (1 \leq i \leq m, 1 \leq j \leq n) \end{array} \cdot$$

(f) $\square$ is called *transition type* and it is an object having a form similar to a Boolean expression with variables – the identifiers of the transition's input places.

The difference of this definition with the definition of the standard GN is in point (d), where two predicates are used.

A GNIA is the ordered four-tuple

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \rangle,$$

where

(a) A is the set of transitions as described above.

(b) π_A is a function giving the priorities of the transitions.

(c) π_L is a function giving the priorities of the places.

(d) c is a function giving the capacities of the places.

(e) f^* is a function that calculates the truth values of the predicates of the transition's conditions. To each element $\langle r_{i,j}^1, r_{i,j}^2 \rangle$ of r^* the function f^* juxtaposes an ordered couple of the truth values of the predicates $r_{i,j}^1$ and $r_{i,j}^2$. This is a new aspect of the GNIA's in comparison to the ordinary GNs.

(f) function $\theta_1 = \bigcup_{Z \in A} \theta_1^Z$, where θ_1^Z is a function giving the next time moment

for which transition Z can be activated, i.e., $\theta_1^Z(t) = t'$, where $t = pr_3 Z$, $t' \in [t, T + t^*]$ and for $1 \leq i \leq n$, $pr_i \{x_1, x_2, \dots, x_n\} = x_i$. The value of this function is calculated at the moment when the transition terminates its current functioning.

(g) function $\theta_2 = \bigcup_{Z \in A} \theta_2^Z$ and θ_2^Z is a function giving the duration of the

active state of a given transition Z , i.e., $\theta_2^Z(t) = t'$, where $t = pr_3 Z \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning.

(h) K is the set of the GN's tokens.

(i) π_K is a function giving the priorities of the tokens.

(j) θ_K is a function giving the time moment when a given token can enter the net.

(k) T is the time moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale.

(l) t^0 is an elementary time step related to the fixed (global) time-scale.

(m) t^* is the duration of the GN functioning.

(n) X is a function which assigns initial characteristics to every token when it enters an input place of the net. If $\alpha \in K$, then it enters the GN with initial characteristic x_0^α .

(o) Φ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition. If $\alpha \in K$, then it, entering an output place of some GN transition and having as current characteristic x_{cu}^α , obtains the next characteristic x_{cu+1}^α .

(p) b is a function giving the maximum number of characteristics a given token can receive.

Algorithm for GNIA functioning. The definition of a transition of a GNIA differs from that of a transition in an ordinary GN. Therefore, a new algorithm for transition functioning is required. We shall propose a general algorithm for transition functioning and an algorithm for the functioning of a GNIA.

The transfer of tokens from the input places to output places of a given transition in GNIA is specified by an algorithm for transition functioning which we denote by *Algorithm A*. It is described below.

(A*01) The input and the output places of the transition are arranged according to their priorities.

(A*02) In each input place two lists are formed: a list of all tokens, arranged by priority, and an empty list.

(A*03) An empty index matrix R^* , which corresponds to the matrix of the predicates r^* of the given transition, is generated. It is initiated with a value “0” (corresponding to value “false”) of all of the elements of this new matrix, which:

- are placed in a row, corresponding to an empty input place;
- are placed in a column, corresponding to a full output place;
- are placed in (i, j) -th position, for which $m_{i,j} = 0$, i.e., the current capacity of the arc between i -th input and j -th output place is zero.

(A*04) The sorted places are passed sequentially by their priority, starting with the place having the highest priority, which has at least one token and through which no transfer has occurred on the current time step. For its highest priority token (from the first list) the predicates corresponding to the relevant row of matrix R^* are checked. The elements of r^* , for which the corresponding elements of R^* are not zeros, are calculated.

(A*05) Depending on the execution of the operator for permission or prohibition of tokens splitting over the net, the token from step A*04 will pass either to all permitted to it output places, or to this very place among them, which has the highest priority. A transfer between the current i -th input place and j -th output place only occurs if the predicate $r_{i,j}^1$ has truth value “true” and the truth value of the predicate $r_{i,j}^2$ is “false”. If the predicate $r_{i,j}^1$ has truth value “true” and the truth value of the predicate $r_{i,j}^2$ is also “true”, then the arc inhibits the token, i.e., the token leaves the net. If one token cannot pass through a given

transition at the current time step, it is moved to the second list of tokens of the corresponding place. The tokens, which have entered into the place after the transition activation, are moved to the second list, too.

(A*06) The capacities of all input for the transition places for which a transfer of a token to output place has occurred or a token has been inhibited by an arc increment by 1 for every token, during the current time step.

(A*07) The capacities of all output for the transition places, which are input for other, active at the moment transitions, increment by 1 for each token that has left them at this time step.

(A*08) The capacities of all output places, in which a token, determined at step **A*04**, has entered, decrement by 1. If the maximum number of tokens for a given output place is reached, the elements of the corresponding column of matrix R are made "0".

(A*09) The capacities of all arcs through which a token has passed or a token has been inhibited decrement by 1. If the capacity of an arc has reached 0, the elements of the corresponding output place of matrix R are given value "0".

(A*10) The values of the characteristic function Φ for the output places, in which tokens have entered (according to step **A*05**) are calculated.

(A*11) If there are more output places for tokens at this step, the algorithm returns back to step **A*04**; otherwise it proceeds to step **A*11**.

(A*12) The current model time t is increased by t^0 .

(A*13) Is the current time moment equal to or greater than $t_1 + t_2$? If the answer of the question is "no", proceed to step **A*04**, otherwise terminate the transition functioning.

After specifying the algorithm for transition functioning in GNIA, we need to introduce an algorithm for GNIA functioning. This algorithm will be referred to as *Algorithm B*. As in the ordinary GNs case, the concept of *Abstract Transition (AT)* should be used. This is a transition which is the union (see [3] for the operation "union" of transitions) of all active transitions of a GN at a given moment of time. The algorithm is described below.

(B01) Put all tokens α for which $\theta_K(\alpha) \leq T$ into the corresponding input places of the net.

(B02) Construct the GN's AT (initially it is empty).

(B03) Check whether the value of the current time is less than $T + t^*$.

(B04) If the answer to the question in **B03** is "no", terminate the GN process.

(B05) Check all transitions for which the first time component is exactly equal to the current time moment.

(B06) Check the transition types of all transitions determined by **B05**. The method of checking is as follows:

- change the names of all places which participate in the Boolean expression of the transition type as variables with values: 0, if the corresponding place has no tokens at the current moment; 1 – otherwise;

- calculate the truth value of the so obtained Boolean expression.

(B07) Add all transitions from **B06** for which the transition types are satisfied by the AT.

(B08) Apply Algorithm A over the AT.

(B09) Remove from the AT all transitions which are inactive at the current time moment.

(B10) Increase the current time by t^0 .

(B11) Go to **B03**.

As in the case of the ordinary GNs, another algorithm can also be used, but the present one describes the processes in more details.

GNIAs are a conservative extension of the GNs. An important aspect of the GNs theory is the question whether the extensions of the ordinary GNs are conservative. In order to study this problem for the GNIA, let Σ_{IA} be the class of all GNIAs and Σ – the class of all standard GNs. We must mention that here we use the word “class” instead of “set” in the sense of Neuman–Bernais–Gödel’s sets theory. Obviously, every standard GN is a GNIA, i.e., the relation $\Sigma_{IA} \vdash \Sigma$ holds (see [2, 3]). This is so because every standard GN can be represented by a GNIA in which all elements of the IM r^* have the form $\langle r_{i,j}, “false” \rangle$. Therefore, the class Σ_{IA} is an extension of the class Σ of the standard GNs.

We will prove the following theorem.

Theorem 1. *The class Σ_{IA} is a conservative extension of the class Σ , i.e., $\Sigma_{IA} \equiv \Sigma$.*

Proof. To prove $\Sigma_{IA} \equiv \Sigma$, because of $\Sigma_{IA} \vdash \Sigma$, it is enough to show that the relation $\Sigma \vdash \Sigma_{IA}$ holds.

Let E be an arbitrary GNIA:

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \rangle.$$

We will construct an ordinary GN G with the same time components and the same number of transitions which represents the functioning and preserves the results of the work of E .

The theorem for the completeness of the GN transitions will be used. For each transition Z of E we will construct a corresponding transition Z^* of G such that the tokens that enter them through corresponding input places with the same characteristics go to the corresponding output places with equal characteristics.

Let G be an ordinary GN with components:

$$G = \langle \langle A^*, \pi_A, \pi_L^*, c^*, f^*, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi^*, b \rangle \rangle,$$

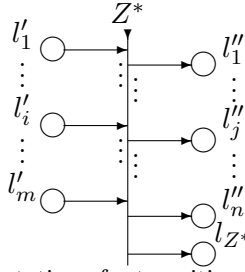


Fig. 2. Graphical representation of a transition of an ordinary GN

where A^* is the set of the transitions of G . Let $Z = \langle L', L'', t_1, t_2, r, M, \square \rangle$ be an arbitrary transition of E . We will construct a transition Z^* (see Fig. 2) of G corresponding to transition Z in the following way: $Z^* = \langle L', L''^*, t_1, t_2, r^*, M^*, \square \rangle$, where $t_1, t_2, L', M, \square$ are as above and $L''^* = L'' \cup \{l_Z\}$, i.e., to each transition Z of E we add additional place l_Z , where the tokens that are inhibited by the arcs of Z will be transferred to. Place l_Z has the lowest priority among the places of the transition. Let $pr_i B$ be the i -th projection of the n -dimensional set B . If $r = pr_5 Z = [L', L'', \{\langle r_{l_i, l_j}^1, r_{l_i, l_j}^2 \rangle\}]$ and $M = pr_6 Z = [L', L'', \{m_{i, j}\}]$ have the form of IMs, then $r^* = pr_5 Z^* = [L', L'' \cup \{l_Z\}, \{r_{l_i, l_j}^*\}]$, where

$$(\forall l_i \in L')(\forall l_j \in L'')(r_{l_i, l_j}^* = r_{l_i, l_j}^1), \quad (\forall l_i \in L') \left(r_{l_i, l_Z}^* = \bigvee_{j \in L''} (r_{l_i, l_j}^1 \wedge r_{l_i, l_j}^2) \right).$$

$$M^* = pr_6 Z^* = [L', L'' \cup \{l_Z\}, \{m_{i, j}^*\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(m_{i, j}^* = m_{i, j}), \quad (\forall l_i \in L')(m_{i, Z}^* = \infty).$$

A^* is the set of the transitions obtained by repeating the above procedure for all transitions of E . Function $\pi_L^* = \pi_L \cup \pi_{\{l_Z | Z \in A\}}$, where function $\pi_{\{l_Z | Z \in A\}}$ determines the priorities of the new places that are elements of set $\{l_Z | Z \in A\}$ and the priorities of places l_Z for every transition $Z \in A$ are the minimal among the priorities of all places of the transition Z . Function $c_L^* = c_L \cup c_{\{l_Z^* | Z^* \in A^*\}}$, where the capacities of the new places for every transition are " ∞ ", i.e., they have unlimited capacities. Function $\Phi^* = \Phi \cup \Phi_{\{l_Z^* | Z^* \in A^*\}}^*$, where function $\Phi_{\{l_Z^* | Z^* \in A^*\}}^*$ determines the characteristics obtained by the tokens in the l_Z -places for every transition $Z^* \in A^*$. The tokens entering these additional places l_Z for every transition $Z^* \in A^*$ do not obtain any new characteristic.

We shall prove that both GNs E and G function equally. We shall compare the functioning of one arbitrary transition Z of the GNIA E and its corresponding transition Z^* of G which has been constructed in the way described above.

Obviously, these two transitions become active simultaneously and have equal duration of functioning and priorities. The corresponding places have equal pri-

orities and capacities in both nets and the transition's arcs have equal capacities except for the additional place l_Z of Z^* .

Let us compare the transfer of two arbitrary corresponding tokens with equal characteristics that enter simultaneously two corresponding input places of the two transitions. If these transitions are input ones for their nets, then the two corresponding tokens will enter them with equal characteristics. Let us assume that in some time moment of the GNs functioning all previous characteristics of the two tokens are the same. If the arc through which the token in E is transferred is an inhibitor arc, then according to the algorithm for transition functioning in GNIA, the two corresponding tokens will be either transferred to two corresponding places, or will not be transferred, or the token in E will be inhibited by the arc while the token in G will be transferred to the additional place l_Z of the transition and will leave the net, i.e., the results of the functioning are similar.

Therefore, the two arbitrarily selected tokens behave in the same way, and thus the two arbitrarily chosen corresponding transitions function equally. From the theorem for the completeness of the GN transitions (see [2, 3]), it follows that the two nets function equally. This proves the theorem. $\square$

Conclusion. GNIAs, as a new conservative extension of the class of the ordinary GNs, can be useful for the modelling of real life processes. In particular, in service systems and telecommunication networks, there are cases when the service of requests is suddenly interrupted by the users or due to technical failures of the network. Such reasons for service ending are discussed in the documents of the International Telecommunication Union (ITU). In [7], reasons for service ending such as blocked, suppressed, intended, etc., are specified. GNIAs can be especially useful in the modelling of telecommunication systems where such reasons for service ending are presented.

In future, we shall search for real-life processes where the use of GNIAs for the modelling and simulation would give better results and conceptual clarity.

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