POLOIDAL-COMPRESSIONAL MHD MODES OF THE EARTH’S IONOSPHERE

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Abstract

Large-scale MHD modes supported by the Earth’s ionosphere are considered. The ionosphere plasma and the static Earth’s geomagnetic field are considered in a quasi-dipole box system, where the static magnetic field varies both in direction and in magnitude. A basic wave equation for function $E$ (electric field) is derived for poloidal-compressional MHD modes and subsequently solved taking into account the magnetic field magnitude changes. Resonance frequencies, amplitude profiles and polarization features of the poloidal-compressional MHD modes of the ionosphere are pointed out.

Key words: ionosphere, dipole magnetic field, poloidal-compressional MHD wave and modes, Alfvén velocity, resonance frequency

Introduction. The Earth’s ionosphere represents a thin spherical plasma structure that encompasses the Earth’s atmosphere. It is bounded above by several structures of rarefied plasmas (one above the other). Formerly, the question of MHD modes and/or resonances of the Earth’s ionosphere has been reduced to the so-called Ionospheric Alfvén Resonances (IAR) [1,2,5,8–13]. IARs are low-frequency Alfvén waves propagating vertically and being reflected at points where the ionospheric plasma density gradients are the steepest ones thus forming standing Alfvén wave structures. They have discrete spectrum of frequencies usually in the interval $0.3–3$ Hz. Many observations have confirmed the existence of such MHD modes of the ionosphere [2–11].

The fact of existence of IAR events presumes that the Earth’s ionosphere (assuming it as a cavity/waveguide) can support another type of MHD waves, for

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example, MHD surface waves of large enough spatial scales. What large scales? The topside ionosphere is characterized by a rapidly increasing Alfvén speed with a scale height the order of 1000 km (e.g. \cite{10,13}). The bottomside ionosphere (definitely below the \( E \) region) undergoes plasma density decrease (and Alfvén velocity increase even to infinity) on much smaller scales. Thereby, MHD surface waves can be supported by the ionosphere plasma provided that their spatial scales exceed \( 10^3 \) km. In this case, the Earth’s ionosphere should be considered a thin structure of spherical shape concentric to the Earth surface. MHD waves of planetary (or global scales, scales comparable to the Earth radius, \( R_E \)) then could be considered possible modes of the Earth’s ionosphere. Accordingly, the question about the existence of large-scale MHD modes supported by the ionosphere plasma as a cavity/waveguide is quite reasonable.

The main problem of large-scale MHD waves in the Earth’s ionosphere is rooted in the fact that the geomagnetic field \( B \) varies both in direction and in magnitude. It is a horizontal plane (at the equator) and oriented in vertical direction at the geomagnetic poles. The geomagnetic field magnitude also varies, at the poles it is nearly two times greater compared to its magnitude at the equator. On the other hand, existence of large-scale MHD waves in bounded systems has been proven in many cases.

So, the question of the existence of large-scale MHD modes supported by a system of bounded in size plasmas permeated by the static magnetic field of both changeable direction and magnitude is still an open/unsolved plasma physics problem.

In this study, the following task should be questioned/considered: whether the Earth’s ionosphere permeated by the Earth magnetic field of changeable direction and magnitude can support MHD modes of scales comparable to the Earth magnetic field scales. Such MHD modes should propagate or oscillate throughout all latitudes (the ionosphere waveguide).

**Theoretical model.** The ionospheric plasma occupies a region \( R_i - h/2 \leq r \leq R_i + h/2 \), where \( R_i \) is the central line of the ionosphere and \( h \) is the ionosphere thickness. The ionosphere plasma density is assumed to vary in height nearly uniform along \( \theta \) coordinate. The geomagnetic field that permeates the ionosphere region is modelled by a magnetic dipole of axial symmetry. Under a constant height (constant radial coordinate \( r \)), the magnetic field is dependent only on the \( \theta \) coordinate. The magnetic field dipole components are given by

\[
B_{0,r} = -2B_{eq} \cos(\theta), \quad B_{0,\theta} = B_{eq} \sin(\theta), \quad B_\phi = 0,
\]

where \( B_{eq} = B_{eq}(r) \) is the static magnetic field at the magnetic equator. Hence, it varies strongly in direction and slowly in magnitude (up to 2 times) at poles (\( \theta = 0, \pi \)). For the Earth’s ionosphere, the geomagnetic field magnitude at the equator is around 0.3 G (1 G = \( 10^{-4}T \)) while at the poles \( B_0 \) is around 0.55 G.
We study the problem of possible existence of large-scale MHD wave modes supported by ionosphere (plasma cavity/waveguide) immersed in a dipole magnetic field. The plasma motion thus is governed by the equations of the ideal magnetohydrodynamics. The ionospheric plasma waveguide is considered cold plasma, then its sound speed \( c_0 = \gamma p_0/\rho_0 \) is much less than the Alfvén velocity

\[
v_A = \sqrt{\frac{B_0^2}{\mu_0 \rho_0}}, \quad v_A \gg c_0.
\]

As at ionospheric heights the sound speed \( c_0 \) is definitely less than \( 10^4 \) m/s (the Alfvén velocity \( v_A \) is around \((1–3) \times 10^5\) m/s), we will neglect kinetic pressure effects \((p_0 \rightarrow 0)\). A quantity \( E \) (electric field) equal to

\[
E = -(v \times B_0)
\]

is introduced. Quantity \( E \) coincides with the induced electric field when the plasma conductivity is infinite. Multiplying the motion equation (in ideal magnetohydrodynamics) with \( B_0 \) we get:

\[
\mu_0 \rho_0 \frac{\partial E}{\partial t} = (B_0 \cdot j)B_0 - (B_0^2)j.
\]

After time derivative of both sides of (4) and the Maxwell equation

\[
\nabla \times b = \mu_0 j
\]

a generalized form of wave equation governing MHD disturbances in a system of cold plasma immersed in varying in direction static and inhomogeneous static magnetic field \( B_0 \) is obtained:

\[
\mu_0 \rho_0 \frac{\partial^2 E}{\partial t^2} - B_0^2 (\Delta E - \nabla \nabla \cdot E) + \left( B_0 \cdot \frac{\partial j}{\partial t} \right) B_0 = 0.
\]

Basic equation (6) comprises all MHD modes of the ionosphere plasma immersed in dipole magnetic field. In a special case of MHD wave disturbances of axial symmetry, MHD wave quantities (plasma velocity \( v \), magnetic field disturbances \( b \)) may be separated in poloidal-compressional \( (v_r, v_\theta; b_r, b_\theta) \) and toroidal \( (v_\phi, b_\phi) \) ones. Studying poloidal-compressional modes (assuming axial propagation only), terms \( B_0 \cdot j \) and \( \nabla \nabla \cdot E \) are simply zeroed. The velocity disturbance \( v \) then is coplanar with the \((r, \theta)\) plane, while the electric field \( E \) is directed along the azimuthal, \( \phi \)-axis. Equation (6) is reduced to

\[
\mu_0 \rho_0 \frac{\partial^2 E_\phi}{\partial t^2} - B_0^2 \Delta E_\phi = 0,
\]
where $B^2_0(\theta)$ is the squared magnitude of the dipole magnetic field depending on $\theta$:

$$B^2_0(r, \theta) = (3 \cos^2 \theta + 1)B^2_{eq} \equiv f(\theta)B^2_{eq}(r).$$

Here $\theta$ is the polar angle, $B_{eq}$ stands for the dipole magnetic field magnitude at equator ($\theta = \pi/2$). The factor

$$f(\theta) = 3 \cos^2 \theta + 1$$

incorporates the dipole magnetic field dependence on polar angle $\theta$ at constant radial distance $r$.

**Analysis.** In essence, eq. (7) represents a wave equation for the chosen function $E(3)$. Actually, the ionosphere can be modelled as a spherical cavity/waveguide of thickness $h$ that is much less than the Earth radius $R_E$ ($h \ll R_E$). For convenience, we adopt 2D Cartesian frame of reference to eq. (7) (quasi dipole box model) for a preliminary study of possible large-scale MHD modes of the ionosphere-dipole magnetic field system. Then we look for solutions in the form:

$$E_\phi = A(x)E(y, z) \exp(i\omega t),$$

where new variables $x$ and $(y, z)$ stand for $r$ and $(\theta, \phi)$ coordinates, respectively. Variables $r, \theta$ and $x(r), y = r\theta$ replace each other. The magnetic field anisotropy factor (9) is accounted for through a simple relation: $\theta \equiv y/r_i$ ($y = r_i\theta$). Assume the following reasonable $r(x)$ dependence

$$A_i(x) = \text{const}, \quad r_i - h/2 \leq x \leq r_i + h/2$$

and

$$A_+(r) = A_i(r = r_i + h/2) \exp(-\kappa(x - (r_i + h/2))), \quad x \geq r_i + h/2,$$

$$A_-(r) = A_i(r = r_i - h/2) \exp(-\kappa(x - (r_i + h/2))), \quad x \leq r_i - h/2.$$

The attenuation coefficient $\kappa$ appearing in (12) is to be determined from the $\nabla \cdot \mathbf{b} = 0$ condition. We seek eigenfunctions and eigenvalues of eq. (7). For axially symmetric MHD compressional modes, the azimuthal number $m$ is equal to zero, wave equation (7) then reads

$$\frac{d^2E(\theta)}{d\theta^2} + \frac{\lambda^2}{3\cos^2(\theta) + 1}E(\theta) = 0,$$

where dimensionless parameter $\lambda^2$ stands for

$$\frac{\mu_0\rho_0\omega^2}{B^2_{eq}}r_i^2 = \frac{\omega^2}{v_A r_i^2}.$$
where $\omega$ and $B_{eq}$ stand for the MHD mode frequency and geomagnetic field magnitude at equator ($\theta = \pi/2$), respectively. In order to find corresponding eigenfrequencies the following boundary conditions are set: Initial values of function $E$ are

$$E(\theta = 0) = 0 \quad \text{and} \quad \frac{dE(\theta = 0)}{d\theta} = 1,$$

where $\theta$ lies in the range $(0, \pi)$. Solutions need to fulfil the (resonance) condition, e.g. $[3,11]$. In our case it sounds

$$E(0) = E(\pi) = 0.$$  

Notice that eqs (13) and (15) govern large-scale poloidal-compressional MHD waves of axial geometry and yield a continuum frequency spectrum. Condition (16) holds solely for MHD modes of discrete frequency spectrum (resonances). Numerical calculations of eq. (13) together with (15) and (16) are performed. Method of Adams/BDF (Backward Differentiation Formulas) has been applied. Figures 1 and 2 illustrate amplitude profiles of the field and velocity (poloidal and compressional components) of the first MHD modes (fundamental, $\lambda = 1.21$, \(\lambda = 1.21\).
Fig. 2. First harmonic of the MHD mode. Calculated amplitude profiles of incident and reflected (dotted lines) electric field $E(\theta)$ and velocity (poloidal $V_r(\theta)$ and compressional $V_\theta(\theta)$ components) in range of $\theta = 0 \div \pi$

and first harmonic, $\lambda = 2.94$). Dotted lines there trace the amplitude profiles of reflected poloidal-compressional MHD mode. For example, note a phase reversal of velocity component $V_\theta$ of the fundamental mode which occurs through point $\theta = \pi/2$, $V_\theta$ amplitude peaks are at polar angles $\theta$ approximately equal to $68^\circ$ and $112^\circ$ or at around $\pm 22^\circ$ in magnetic latitude. The electric field and the other velocity component have amplitude peaks at equator. A common feature of the modes is that the electric field and velocity components are practically zero at and around the pole zones ($\theta \approx 0$, $\theta \approx \pi$).

The MHD mode resonance frequencies are easily estimated from the eigenvalues $\lambda_n$, $n = 1, 2$, of eq. (13). Using expression (14), the first (fundamental) resonance frequency $f_1$ is estimated:

(17) \[ f_1 \approx 9.27 \sqrt{\frac{N_{m,0}}{N_m F_2}} \text{ mHz.} \]

It is inverse proportional to the squared F2 peak electron density $N_m F_2$. $N_{m,0}$ is a constant (being) fixed to $2.5 \times 10^{11}$ m$^{-3}$. Notice the $N_{m,0}$ value corresponds to an ionosphere critical frequency of 4.5 MHz. In estimation (17), the radial distance $r_i$ is taken equal to $r_i \equiv R_E + h/2 \approx 6850$ km. The next resonance frequency of

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poloidal-compressional MHD mode of the ionosphere ($\lambda_2 = 2.94$) is

\[ f_2 = 22.5\sqrt{\frac{N_m \theta}{N_m F_2}} \text{ mHz}. \]

(17)

Notice that the Alfvén velocity magnitude of the poloidal-compressional MHD modes of the ionosphere depends on the plasma density of the F2 region $N_m F_2$. In daytime, regular density $N_m F_2$ increases to and above $10^{12}$ m$^{-3}$. This suggests in daytime, the resonance frequencies $f_1$ (or $f_2$) decrease to 4.6 (or 11.2) mHz. Regular density $N_m F_2$ variations (local time variations) are accompanied by seasonal variations, solar and geomagnetic activity effects [14–16] and even impacts through the neutral atmosphere of global scales [17]. They will be subject of further research.

One of the important aspects of consideration of large-scale MHD modes is the mode polarization. The solution of eq. (12) is easily transformed into expressions for the wave plasma velocity $v$ and magnetic field $b$. Under axial propagation conditions the plasma velocity and the magnetic field components are completely in the meridional, $(r, \theta)$ plane (poloidal modes). Expressions for the plasma velocity are given:

\[ v_r = \frac{E_\theta(\theta) \sin \theta}{B_{eq}(r_i)(3 \cos^2(\theta) + 1)}, \quad v_\theta = -2 \frac{E_\phi(\theta) \cos \theta}{B_{eq}(r_i)(3 \cos^2(\theta) + 1)}. \]

Main polarization features of the MHD modes of the ionosphere are mutual rotation of the wave velocity and associated magnetic field components in $\theta$ (in meridional plane). Corresponding phase reversals emerge when crossing the equator.

**Discussion and conclusion.** Mode analysis of the derived basic MHD equation reveals a possible existence of large-scale poloidal-compressional MHD modes supported by the Earth’s ionosphere waveguide. It is worth noting that the poloidal-compressional MH-mode-field amplitude evanesces away from the ionosphere with coefficient $\kappa$ (12). Coefficient $\kappa$ is equal to the poloidal wavenumber $k_\theta$ given by

\[ k_\theta = \frac{2\pi n}{\pi r_i} = \frac{2n}{r_i}, \quad n = 1, 2, \ldots \]

Having in mind that radius $r_i = R_E + h/2$ exceeds the Earth atmosphere thickness (including the lower atmosphere) of about 250–300 km, it follows that the poloidal-compressional mode field penetrates to the Earth surface with ignorable changes in amplitude. MHD mode penetration through the lower part of the ionosphere should initiate Hall currents there (in daytime). As a result, the Hall current associated magnetic field will induce an azimuthal magnetic field component on the Earth’s surface. Consequently, ground-based “projection” of large-scale MHD mode fields should appear in both meridional plane and azimuthal directions.
The magnetic field anisotropy factor \( f(\theta) \) is an essential factor that governs the spectrum of the poloidal-compressional MHD modes in the Earth ionosphere. If the anisotropy factor in \( f(\theta) \) was ignored (setting \( f(\theta) \to 1 \)), basic wave equation (7) would be similar to that of the Schumann resonances \([12]\) except the characteristic velocity (Alfvén velocity \( v_A \) stands for light velocity \( c \)).

The basic wave equation (7) implies an extensive study of MHD modes of the ionosphere cavity/waveguide of arbitrary plasma densities. An “anisotropy” function of general form \( f(B(\theta), n(\theta)) \) then will give an account of the density profiles of arbitrary dependence on angle \( \theta \) (based on empirical and/or model profiles, e.g., IRI).

Field-line resonances (FLR) and MHD cavity modes are well-known phenomena of the Earth’s magnetosphere. In the inner magnetosphere (plasmasphere) their frequencies are in the Pc3 and Pc4 pulsation diapasons: 6.7–22 mHz and 22–40 mHz. MHD cavity modes of the plasmasphere considered a source of FLRs are with the same frequencies. In particular, poloidal MHD modes of the plasmasphere are standing MHD waves bouncing (in radial direction) between the ionosphere and the plasmapause \([4,10]\). The latter have been considered most probable sources of Pc3–4 and Pi2 pulsations characterized by common frequency spectrum at both mid- and low-latitudes.

The poloidal-compressional MHD modes of the ionosphere found in this study consist of fundamental mode and harmonics and their frequency spectrum (17) and (17′) belong obviously to the Pc3–4 diapason. Appearance of poloidal-compressional, \((r, \theta)\), components of the ionospheric MHD mode, their spatial structure in latitude and their additional polarization changes through the lower ionosphere and atmosphere toward the ground, all these features are, however distinct from the aforementioned poloidal MHD modes of the plasmaspheric cavity. The poloidal-compressional MHD of the ionosphere are definitely large-scale structures of global scales. Their fields oscillate along latitudes rather than in radial direction. The spatial extent of the poloidal-compressional MHD modes spreads to higher latitudes, encompasses both the plasmapause and high-latitude ionosphere.

The specificity of their field structure is:

1. All poloidal-compressional MHD modes have nodes at the magnetic poles;
2. Higher harmonics have additional nodes emerging at equator and/or certain latitudes between the poles and equator;
3. Except the fundamental mode, the peak amplitude of the harmonics are not symmetrical to the equator, e.g. the peak amplitude of the first harmonics, \( f_2 \), is located at polar angle of \( \sim 52^\circ \) (38° magnetic latitude).

The latitude location of poloidal-compressional MHD modes of the ionosphere also varies for each harmonic. Therefore, it can be expected that different location of ionosphere MHD modes with discrete frequencies will be observed. And vice
versa, the localization zones of the ionospheric MHD harmonics will alternate with latitude zones of negligible and/or zero amplitude.

In conclusion, the poloidal-compressional MHD modes of the ionosphere are completely different from the FLR and cavity modes. First, amplitude localization of the MHD modes of the ionosphere (fundamental and harmonics) is quite different. The fundamental mode has amplitude peaks at the equator, the first harmonic – close to mid latitudes. Nodes of MHD modes of the ionosphere of different discrete frequencies appear at different latitudes, which suggests that at these latitudes the ionospheric MHD modes may not be recorded. All the MHD modes of the ionosphere have nodes (and negligible amplitudes) at (near) the magnetic poles. The MHD modes of the ionosphere and their discrete spectrum seems not to be easily measured and recognized. Simultaneous and multipoint measurements embracing all latitudes (including low-, mid- and high-latitudes) are prerequisite for their observation and identification. These poloidal-compressional MHD modes of planetary scales bring out the range of the global MHD modes of the Earth’s magnetosphere [18] to include MHD modes of the Earth’s ionosphere, as well.

Summarizing one may conclude the following:

Large-scale poloidal-compressional MHD modes of the ionosphere can be initiated by various mechanisms ranging from:

1. Processes occurring at the Earth’s surface (Rayleigh seismic waves, tsunami) and/or by atmospheric winds (in the ionosphere) in which the neutral atmosphere is serving as a mediator;
2. Large-scale electric currents flowing in the lower ionosphere (E region);

Supposedly, ground-based and satellite magnetic field recordings of discrete spectrum (in the mHz diapason: 5–100 mHz) at low- and mid- to high-latitudes, specific field amplitude profile, phase and polarization characteristics of each mode frequency in this spectrum, are prerequisite for an identification and discrimination of the large-scale (global) MHD mode nature, their origin (carriers) and sources.

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