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A NEW EMPIRICAL RELATION CONVERTING MAGNITUDE APPLIED IN BULGARIAN SEISMOLOGICAL PRACTICE (M_P) TO MOMENT MAGNITUDE (M_W)

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Abstract

Earthquakes are some of the deadliest natural disasters affecting human environment. The kinematic and dynamic parameters of earthquakes are the basis of almost all research in the Earth sciences. The results of theoretical and applied seismology are used to explain the main social and economic problems associated with earthquakes and their consequences. Knowledge of the earthquake source parameters enables us to understand better the recent tectonic setting and earthquake generation processes. One usable method for estimating source parameters of earthquake is the spectral analysis of seismic waveforms. The earthquake source parameter – seismic moment, M_0, has the largest practical and scientific application. From M_0 seismic moment magnitude M_W (that is a measure of the energy released by an earthquake) can be computed. In the present study, we aim to improve the scaling between M_P (local magnitude applied in Bulgarian seismological routine practice) and M_W. We extend the M_W estimates to lower magnitudes via a spectral fitting method. Applying P-wave displacement spectra we estimate M_W for 127 small to moderate earthquakes with magnitude ranging between 2.0–5.3 which occurred in Bulgaria and surroundings from 2008 to 2024. The considered earthquakes are recorded at stations of Bulgarian Seismological Network – NOTSSI. As a result, we propose a new empirical scaling relation between local magnitude M_P and seismic moment magnitude M_W. Additionally, 148 M_W estimates from the seismological centres – ISC, EMSC and Harvard are used. The well-defined relation between

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M_W and M_P can be accurately used for compiling homogeneous, with respect to magnitude, earthquake catalogue for Bulgaria.

Key words: earthquake, seismic moment, seismic moment magnitude, spectral analysis

Introduction. The magnitude describes the energy of the seismic event and the dynamics of the fracturing processes.

The first magnitude scale was developed by RICHTER [1]. The scale describes earthquakes in California and relates the distance to the earthquake to the amplitude of the waveform recorded on the Wood–Anderson seismometer. The "Richter Scale" allows the strength of earthquakes to be compared in an instrumental way on a global scale.

At present, magnitude is the most widely used measure of the earthquake strength and has become a ubiquitous parameter in public consciousness.

A large variety of magnitude scales has been proposed over the years. Most of the magnitude scales used the measurement of the amplitudes on the seismogram. Large variations in the size of amplitudes can cause a substantial difference in the magnitude estimates, due to the position of the station relative to the source and local variations in the attenuation of the seismic waves. The magnitude scales being empirical, do not provide information about the physical properties of the earthquake source and moreover they exhibit saturation effects for strong earthquakes.

In the second half of the 20th century a new magnitude scale was defined – the seismic moment magnitude scale – M_W [2,3]. To estimate the moment magnitude, M_W, it is necessary to determine the seismic moment (M_0) of the earthquake.

The seismic moment M_0 is a physical quantity that is a measure of earthquake size and is applied to earthquake moment magnitude assessment. The seismic moment M_0 is estimated either by moment tensor inversion or by spectral analysis of the observed waveforms. M_0 is defined from the amplitude of the flat part of the source spectrum of the P, S, or Lg waves.

In seismological research it is assumed that the moment magnitude correctly estimates the size of earthquakes, as it is determined by the size of the fault and the dislocation along it. M_W scale does not saturate, since it is directly proportional to the logarithm of seismic moment. M_W usually estimates for events above a selected threshold.

Since the M_W scale is presented, many studies are carried out in different regions to create relations between other magnitude scales to M_W [4, 5].

To expand the obtainable magnitude range for empirical ground motion relation, the direct definition of M_W for small events is essential [6].

The most important information for earthquake source parameters is derived from seismic wave spectral analysis. The seismic moment M_0 is estimated applying BRUNE model [7]. M_W can be computed from seismic moment using relation presented in Hanks and Kanamori [3]. The moment tensor inversion method is more complicated and cannot provide a magnitude estimate based on a single station. The spectral analysis of seismic waveforms gives reliable magnitude estimate related to physical quantity and can estimate magnitude based on only one station.

In the present study the empirical relations converting magnitude $\mathrm{M_P}$ used in Bulgarian seismological routine practice (defined by Christoskov [8]) to moment magnitudes $\mathrm{M_W}$ (seismic moment magnitude defined by displacement spectra generated for P-waves) are derived. The results are compared with $\mathrm{M_P/M_W}$ relations that are presented in [9].

The work detailed in this article is to improve and extend to lower magnitudes the relations between local magnitude $\mathrm{M_P}$ and seismic moment magnitude $\mathrm{M_W}$.

Data and method. In the present study, we improve the relations between $\mathrm{M_P}$ (local magnitude) and $\mathrm{M_W}$ (seismic moment magnitude) by applying P-wave displacement spectra. To define the empirical relations a sample of data from the Bulgarian seismological network (NOTSSI) has been collected and processed. The compiled data set contains 275 earthquakes with magnitude $\mathrm{M_P}$ ranging between 2.0–5.9 which occurred in Bulgaria and surroundings from 2008 to 2024 (Fig. 1). The body P-wave magnitude $\mathrm{M_P}$ that is applied in the routine practice of Bulgarian seismology is presented in [8].

In the analysis two data samples are considered – the first one includes 127 earthquakes with magnitude $\mathrm{M_P}$ ranging between 2.0–5.3 and $\mathrm{M_W}$ that is estimated using a spectral fitting method; the second data sample includes 275 earthquakes with magnitude $2.0 \leq \mathrm{M_P} \leq 5.9$ (127 $\mathrm{M_W}$ estimates based on P-wave displacement spectra from the data sample 1 and 148 $\mathrm{M_W}$ estimates from international centres – ISC [10, 11], EMSC [11] and Harvard [12]).

The moment magnitude for 127 earthquakes recorded at NOTSSI stations is computed from seismic moment $\mathrm{M_0}$ using the relation of Hanks and Kanamori [3]:

$$\mathrm{M_W} = 2/3 \log \mathrm{M_0} - 10.7. \tag{1}$$

The seismic moment is defined as:

$$\mathrm{M_0} = \mu \mathrm{SD}, \tag{2}$$

where D is the average displacement over the entire fault surface (in m), S is the area of the fault surface (in $\mathrm{m^2}$), and μ (in Pa, or $\mathrm{N/m^2}$) is the average shear rigidity of the faulted rocks.

In the present study the spectra are based on the records at the stations located at a distance less than 200 km.

The Brune model is the most frequently used source model. The earthquake parameters are determined using the spectral level, which can be estimated by fitting a source model spectrum to amplitude spectra [14].

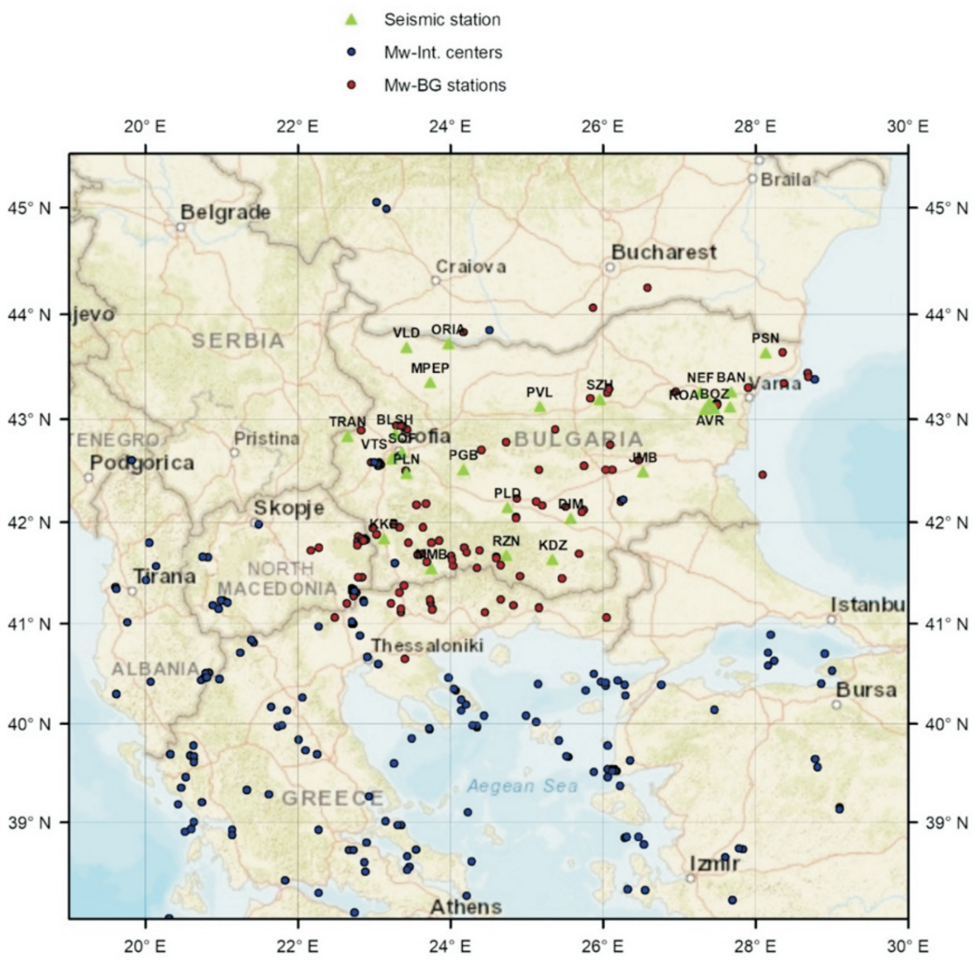

Fig. 1. Seismicity in and near Bulgaria (instrumental earthquakes with $M_P \geq 2.0$ which occurred from 2008 to 2024)

In the research, the seismic moment M_0 for P-waves is calculated using the following relationships:

$$M_{0P} = \frac{\rho 4\pi \Omega_P R v_P^3}{R_{\theta\varphi}(P)}, \tag{3}$$

where ρ is the density in g/cm^3, Ω_P is the spectral level in nm*s, v_P is the velocity of P-wave in km/s, R is the distance in km, $R_{\theta\phi}(P)$ is the radiation pattern.

Aki and Richards [15] presented average value of the correction for P-wave radiation pattern of about 0.52.

The so-called distance effect (the low frequency spectral amplitude of the P and S wave attenuation) is accounted by the quality factor Q. In the present study, regional attenuation effects are fixed for Q = 400 [16].

As a reference magnitude the body P-wave magnitude $\mathrm{M_P}$ (presented in [8]) is used. The $\mathrm{M_P}$ magnitude is defined using the formula:

$$\mathrm{M_P} = \log\left(\frac{\mathrm{A}}{\mathrm{T}}\right)_{\max} + \sigma_{\mathrm{BB}}(\Delta) + \mathrm{s}, \tag{4}$$

where, $\left(\frac{\mathrm{A}}{\mathrm{T}}\right)_{\max} = \left(\frac{\mathrm{V_{max}}}{2\pi}\right)$ and A is amplitude in µm, T is period in s, and $\mathrm{V_{max}}$ is the peak ground velocity in µm/s of P-phase recorded on the broadband seismograph vertical-component at epicentral distances less than 10°; $\sigma_{\mathrm{BB}}(\Delta)$ is the calibration function; and $\mathrm{s_j}$ is the j station magnitude correction.

In the present study, a regression analysis is applied to minimize the mean square deviation using least square method (LSQ) and orthogonal regression method (OR) (which is also known as the total least squares method or error-invariable method). In applied statistics, orthogonal regression is an error-in-variables regression approach, a least-squares data modelling technique that accounts for the observational errors of both the dependent and independent variables [17]. In orthogonal regression, the purpose is to minimize the orthogonal (perpendicular) distances from the data points to the fitted line [17].

Orthogonal regression (OR) adopts the assumption that x_i and y_i are random variables with unknown variances σ_x^2 and σ_y^2, known variance ratio – $\eta = \frac{\sigma_y^2}{\sigma_x^2}$ ($\eta = 1$ in the study).

The selected model is linear regression between two magnitude scales in the considered range:

$$\mathrm{M_1} = \mathrm{a} + \mathrm{bM_2}, \tag{5}$$

where $\mathrm{M_1}$ and $\mathrm{M_2}$ are magnitudes computed by different magnitude scales and a and b are empirical coefficients.

The coefficients are estimated using the LSQ and OR methods, assuming that a sample of n earthquakes with magnitude estimations of the two scales (x_1, y_1), (x_2, y_2), …, (x_n, y_n) are considered (details are presented in [9]).

The linear correlation coefficient r (also referred to as Pearson correlation coefficient) is a measure of the linear correlation between two variables x and y. The correlation coefficient is a measure that determines the degree to which two variable movements are associated. If there is no linear correlation or a weak linear correlation, r is close to 0. It has a value between $+1$ and -1, where 1 is total positive linear correlation and -1 is total negative linear correlation.

The mathematical formula for defining r is:

$$r = \frac{n\sum\limits_{i=1}^{n} x_i y_i - \left(\sum\limits_{i=1}^{n} x_i\right)\left(\sum\limits_{i=1}^{n} y_i\right)}{\sqrt{n\sum\limits_{i=1}^{n} x_i^2 - \left(\sum\limits_{i=1}^{n} x_i\right)^2}\sqrt{n\sum\limits_{i=1}^{n} y_i^2 - \left(\sum\limits_{i=1}^{n} y_i\right)^2}}. \tag{6}$$

Additionally, the coefficient of determination, r^2 (a measure of how well the regression line represents the data, [9]) is estimated. The coefficient of determination varies between $0 \leq r^2 \leq 1$. A correlation greater than 0.8 is generally described as strong, whereas a correlation smaller than 0.5 is generally described as weak.

Results. Figure 2 presents the variation of the seismic moment magnitude $\mathrm{M_W}$ versus magnitude $\mathrm{M_P}$ that is applied in Bulgarian seismological routine. In the analysis the first data sample (127 earthquakes with magnitude $\mathrm{M_P}$ ranging between 2.0–5.3 and $\mathrm{M_W}$ estimated using a spectral fitting method) is used. The diagram in Fig. 2 illustrates the LSQ and OR fit of $\mathrm{M_W}$ versus $\mathrm{M_P}$ for small to moderate earthquakes which occurred in Bulgaria and surroundings from 2008 to 2024.

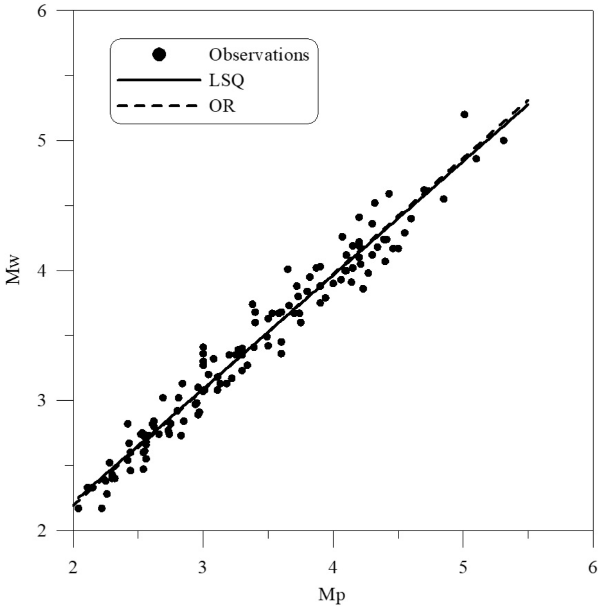

Fig. 2. Correlation between $\mathrm{M_W}$ (127 spectral estimates) and $\mathrm{M_P}$ for small to moderate earthquakes in Bulgaria and surroundings from 2008 to 2024

The relations that express the best-fit line are:

$$
\begin{aligned}
&\mathrm{LSQ}: \mathrm{M_W} = 0.88\mathrm{M_P} + 0.46 \pm 0.14\\
&\mathrm{OR}: \mathrm{M_W} = 0.89 * \mathrm{M_P} + 0.40 \pm 0.14\\
&r = 0.98,\ r^2 = 0.96,\ n = 127,\ 2.0 \le \mathrm{M_P} \le 5.3.
\end{aligned}
\tag{7}
$$

The relation between $\mathrm{M_P}$ and $\mathrm{M_W}$ obviously reveals linear dependency throughout the considered magnitude range $2.0 \le \mathrm{M_P} \le 5.3$. The results indicate a good correlation ($r = 0.98$) between $\mathrm{M_P}$ and $\mathrm{M_W}$, and the uncertainty is reasonable ($\sigma = 0.14$ for both LSQ and OR). The coefficient of determination $r^2 = 0.96$ presents high strength of the linear association between $\mathrm{M_P}$ and $\mathrm{M_W}$ and that the regression lines very well represent the input data.

Figure 3 presents the LSQ and OR fit of seismic moment magnitude $\mathrm{M_W}$ versus magnitude $\mathrm{M_P}$ for small to moderate earthquakes with magnitude $\mathrm{M_P}$ ranging between 2.0–5.9 which occurred in Bulgaria and surroundings from 2008 to 2024. In the analysis the second data sample is used – 275 events: 127 $\mathrm{M_W}$ estimates based on a spectral fitting method and 148 $\mathrm{M_W}$ estimates from international centres.

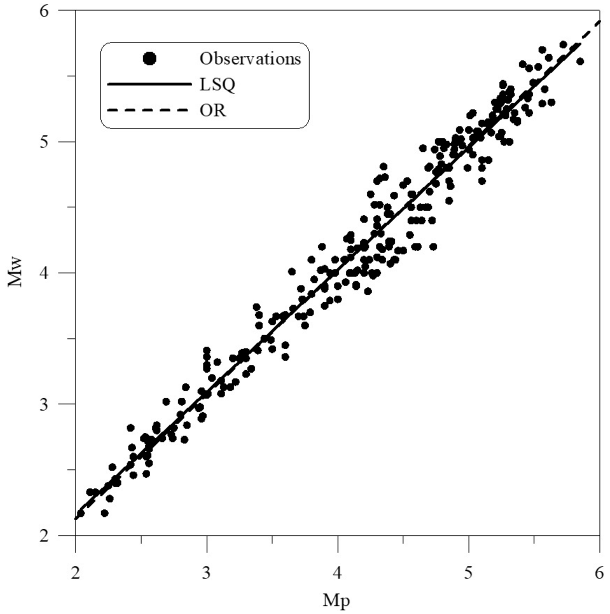

Fig. 3. Correlation between $\mathrm{M_W}$ and $\mathrm{M_P}$ for 275 small to moderate earthquakes in Bulgaria and surroundings from 2008 to 2024

The relations that express the best-fit line are:

$$
\begin{aligned}
&\mathrm{LSQ}: \mathrm{M_W} = 0.93\mathrm{M_P} + 0.29 \pm 0.16\\
(8)\qquad &\mathrm{OR}: \mathrm{M_W} = 0.95 * \mathrm{M_P} + 0.23 \pm 0.16\\
&r = 0.98,\ r^2 = 0.96,\ n = 275,\ 2.0 \le \mathrm{M_P} \le 5.9.
\end{aligned}
$$

The coefficient of determination $r^2 = 0.96$ presents high strength of the linear association between $\mathrm{M_P}$ and $\mathrm{M_W}$ and that the regression lines very well represent the input data.

In the study the derived empirical relations (8) converting magnitude $\mathrm{M_P}$ to moment magnitudes $\mathrm{M_W}$ are compared with those presented in [9]:

$$
(9)\qquad
\begin{aligned}
&\mathrm{LSQ}: \mathrm{M_W} = 0.86\mathrm{M_P} + 0.59 \pm 0.19\\
&\mathrm{OR}: \mathrm{M_W} = 0.93\mathrm{M_P} + 0.31 \pm 0.19
\end{aligned}
$$

The relations are evaluated using different databases. In the present study, the database that is used includes 275 small to moderate earthquakes with magnitude $\mathrm{M_P}$ between 2.0–5.9 which occurred from 2008 to 2024. The $\mathrm{M_W}/\mathrm{M}_P$relation in [9] is determined using a sample of 77 shallow earthquakes with $\mathrm{M_P}$ magnitude ranging from 3.1 up to 5.6 which occurred between 2008 and 2016. The maximum differences in magnitude range 3.1–5.6 between OR estimates are 0.03 for magnitude $\mathrm{M_P} = 5.6$ and for LSQ estimates – 0.092 for $\mathrm{M_P} = 5.6$.

Both relations between $\mathrm{M_P}$ estimates and different $\mathrm{M_W}$ assessments clearly show linear dependency throughout the examined magnitude interval $2.0 \le \mathrm{M_P} \le 5.9$.

Conclusions. Empirical relations converting magnitudes expressed in $\mathrm{M_P}$ magnitude scale (used in Bulgarian seismological routine practice) to moment magnitude $\mathrm{M_W}$ scale, are defined in the study.

Here estimated relations allow routine assessment of $\mathrm{M_W}$ for earthquakes in Bulgaria to be with similar to the $\mathrm{M_P}$ (magnitude used in Bulgarian seismological routine practice) precision.

The results presented here and the need for a magnitude scale that is based on a physical quantity are good arguments for the common use of the manual $\mathrm{M_W}$ estimation.

Reliable $\mathrm{M_W}$ assessments can be a useful tool in producing homogeneous earthquake catalogue – the major input essential for hazard assessment also gives opportunity for correct analysis of dynamic source parameters and estimation of the long-term return rates of damaging earthquakes.

Finally, it should be noted that the presented new scaling relation is purely empirical. The suggested relation in the present study is for practical use in Bulgarian seismological centre. It not only estimates the moment magnitude $\mathrm{M_W}$

more reliably, but also allows further precise relative studies for magnitude unification between the regional networks as NOTSSI and the international seismological centres.

REFERENCES

[1] Richter C. (1935) An instrumental earthquake magnitude scale, Bull. Seism. Soc. Am., **25**, 1–31.

[2] Kanamori H. (1977) The energy release in great earthquakes, J. Geophys. Res., **82**(20), 2981–2987.

[3] Hanks T., H. Kanamori (1979) A moment magnitude scale, J. Geophys. Res., **84**, 2348–2350.

[4] Johnston A. C. (1996) Seismic moment assessment of earthquakes in stable continental regions. Instrumental seismicity, Geophys. J. Int., **124**(2), 381–414.

[5] Di Bona M. (2016) A local magnitude scale for crustal earthquakes in Italy, Bull. Seism. Soc. Am., **106**(1), 242–258.

[6] Cotton F., G. Pousse, F. Bonilla, F. Scherbaum (2008) On the discrepancy of recent European ground-motion observations and predictions from empirical models: analysis of KiK-net accelerometric data and point-sources stochastic simulations, Bull. Seism. Soc. Am., **98**, 2244–2261.

[7] Brune J. (1970) Tectonic stress and spectra of seismic shear waves from earthquakes, J. Geophys. Res., **75**(26), 4997–5009.

[8] Christoskov L., L. Dimitrova, D. Solakov (2012) Digital broadband seismometers of NOTSSI for practical magnitude determinations of P waves, C. R. Acad. Bulg. Sci., **65**(5), 653–660.

[9] Solakov D., S. Simeonova, P. Raykova, I. Aleksandrova (2018) Empirical relation converting Md and Mp magnitudes applied in Bulgarian seismological routine practice to moment magnitude, C. R. Acad. Bulg. Sci., **71**(8), 1076–1085, `https://doi.org/10.7546/CRABS.2018.08.09`.

[10] International Seismological Centre (20XX) On-line Bulletin, `https://doi.org/10.31905/D808B830`.

[11] Dziewonski A. M., T.-A. Chou, J. H. Woodhouse (1981) Determination of earthquake source parameters from waveform data for studies of global and regional seismicity, J. Geophys. Res., **86**, 2825–2852, `https://doi.org/10.1029/JB086iB04p02825`.

[12] EMSC/CSEM, `https://www.emsc-csem.org`.

[13] Ekström G., M. Nettles, A. M. Dziewonski (2012) The global CMT project 2004–2010: Centroid-moment tensors for 13,017 earthquakes, Phys. Earth Planet. Inter., **200–201**, 1–9, `https://doi.org/10.1016/j.pepi.2012.04.002`.

[14] Stork A., J. P. Verdon, J. M. Kendall (2014) The robustness of seismic moment and magnitudes estimated using spectral analysis. Geophysical Prospecting, John Wiley & Sons Ltd. on behalf of Eur. Ass. of Geosc. & Eng, 862–878, `https://doi.org/10.1111/1365-2478.12134`.

[15] Aki K., P. Richards (2002) Quantitative seismology, Second Edition, ISBN 0-935702-96-2, Sausalito, University Science Books, 704 pp.

[16] Malagnini L., R. B. Herrmann, K. Koch (2000) Regional ground motion scaling in Central Europe, Bull. Seism. Soc. Am., **90**, 1052–1061.
[17] Markovsky I., S. Van Huffel (2007) Overview of total least-squares methods, Signal Processing, **87**(10), 2283–2302.

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