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FREQUENCY-WAVENUMBER ANALYSES BY STEERED RESPONSE POWER PHASE TRANSFORM ALGORITHM

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Abstract

This paper is devoted to borrowing some knowledge of related field that deals with array data processing to seismology. Steered-response power with phase transform (SRP-PHAT) is a modern signal processing technique that is treated here for application of small-aperture seismic arrays. It was developed in acoustics for effective sound source localization under strong noise and reverberant conditions, which are absolutely similar to those found in seismological applications and even more complex due to wave propagation through non-homogeneous media. Mathematical representation of this method is given to solve the problem of frequency-wavenumber spectrum density estimation (F-K analysis). The experiments with real seismic array data are presented for two different data sets: The first set belongs to SPITS array data and contains records of P and S phases of the well-known earthquake from 6 October 2006. The second set belongs to temporary array, installed for monitoring and tracking tremor sources. It was shown that SRP-PHAT is a worthy alternative to conventional broadband F-K analysis method involved in most of free and commercial seismological software.

Key words: array signal processing, slowness estimation, tracking tremor sources, phase transform, small-aperture array

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Introduction. Antenna arrays allow to collect data in digitized form by recording specific wavefield generated by a finite number of signal sources. Recorded discrete time series could be used then for further interpretation of observed physical process and estimation of practically important parameters by array signal processing techniques. Today antenna arrays are widely used in such applications as acoustics, seismology, radio astronomy, radar and sonar systems [1–3]. Often, regardless of the application, the model antenna array observations are represented as informative signals (generated by sources) distorted by additive interfering noise. Hence arises mathematical statistical problem to find robust estimator for accurate estimation of unknown parameters of wavefield. In this sense it is always usefull to track and adopt efficient developments of array signal processing from related areas of knowledge.

The phase algorithm SRP-PHAT introduced in [4, 5], was developed for accurate sound source localization and has proven to be one of the most robust to ambient noise and reverberations which are the main cause of distortion of the informative signal. The experience of using phase algorithms in various fields of information technologies, such as communication and acoustics, shows their robustness to characteristics of interfering noise and to deviations of supposed parameters of the wave propagation medium from the real ones. These deviations appear while processing observations in the form of a "signal generated by noise". The phase algorithms suppress (averaging) this "noise" more efficiently than the algorithms using the initial observations. This fact is very important for applications of small-aperture seismic arrays as well, because observed signals at the small-aperture (usually 0.5–3 km) array sensors from seismic source are corrupted by natural or man-made noise or by coda of some seismic signal. It should be noticed that SRP-PHAT belongs to the so-called beamforming techniques, so it could be used only when observed signals from point source at the array sensors are coherent. For small-aperture seismic arrays due to sufficiently short distances between array sensors the observed signals are coherent and SRP-PHAT is applicable to estimate important parameters. In this paper we will consider only small-aperture seismic arrays, therefore the term "small-aperture" will be omitted.

Far-field sources in seismology are considered enough remote events to assume that registered by array sensors seismic signals have plane wavefront. Coordinates estimation problem of such events occurs in 1) monitoring and tracking of volcanic tremor sources [6]; 2) monitoring of underground nuclear tests [7]; 3) determining coordinates of regional event like earthquakes or industrial explosions [8]. The practice and common methodology of using modern automatic seismic monitoring systems for the above-mentioned problems shows that event location uncertainty can be significantly reduced by employing the information on the arrival directions of seismic waves coming at the seismic array sensors, i.e. it is necessary to evaluate their slowness and backazimuth with sufficient accuracy. It is known that such accuracy can be provided by applying standard broad band f-k (BFK)

analyses (frequency-wavenumber) to records of seismic arrays [9, 10], rather than polarization analyses of records of single three-component sensor. But still the problem of monitoring of low-magnitude seismicity is opened for findings of alternative array signal processing techniques that could help to improve accuracy and resolve uncertainties of estimation of slowness vector in relation to conventional methods. For this purpose, SRP-PHAT was applied to real seismic array records for estimation of plane wave slowness vector and the obtained results were compared with existing ones.

Mathematical model of observations and SRP-PHAT estimator. The observed seismic waves propagated through earth medium from source to seismic antenna array can generally be described as outputs of multidimensional linear system with one input and several outputs [11, 12]:

$$\mathbf{y}_k = \mathbf{h}_k(\boldsymbol{\theta}) * s_k + \boldsymbol{\xi}_k, \quad k \in \overline{1,n}, \tag{1}$$

where s_k is the source time function; $\mathbf{h}_k(\boldsymbol{\theta}) = (h_{k,1}(\boldsymbol{\theta}), \ldots, h_{k,m}(\boldsymbol{\theta}))^T$ is the impulse response function, described wave propagation through solid medium between seismic source and array sensors; $\mathbf{h}_k * s_k$ is the convolution of the time series; The vector time series $\mathbf{y}_k = \mathbf{y}(k/f_d)$ are discrete observations of seismic array, f_d is the sampling frequency; $\boldsymbol{\xi}_k = (\xi_{k,1}, \ldots, \xi_{k,m})^T$ are Gaussian stationary processes with a zero vector of average values. It is assumed that noise components are incoherent between any pair of sensors. $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \boldsymbol{\Theta} \subset \mathbb{R}^2$ is the unknown vector parameter of linear system to be estimated.

Due to practical reasons it is convenient to transform discrete observations $\mathbf{y}_k$, $k \in \overline{1,n}$ into the frequency domain by discrete finite Fourier transform (DFFT): $\mathbf{y}_k \Leftrightarrow \dot{\mathbf{y}}_j$, where

$$\dot{\mathbf{y}}_j = (\dot{y}_{j,1}, \ldots, \dot{y}_{j,m})^T = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \mathbf{y}_k \exp\left\{\frac{-2i\pi jk}{n}\right\}, \quad j = \overline{1,n}. \tag{2}$$

Then according to similar DFFT transformations we have $\mathbf{h}_k(\boldsymbol{\theta}) \Leftrightarrow \dot{\mathbf{h}}_j(\boldsymbol{\theta})$, $s_k \Leftrightarrow \dot{s}_j$, $\boldsymbol{\xi}_k \Leftrightarrow \dot{\boldsymbol{\xi}}_j$ and the model of array observation (1) in the frequency domain can be written as follows:

$$\dot{\mathbf{y}}_j = \dot{\mathbf{h}}_j(\boldsymbol{\theta})\, s_j + \dot{\boldsymbol{\xi}}_j. \tag{3}$$

The most effective computational procedures for averaging noise in problems of determining the parameters of observed signals are synthesized by methods of mathematical statistics: for given probabilistic parametric observation models, statistical estimation theory makes it possible to find estimates of signal parameters that are the best in terms of a certain quality criterion, for example, the standard deviation of the estimate from the true value of the parameter. In this

sense model (3) can be used to derive phase-based SRP-PHAT estimator:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \sum_{j=1}^{n} \left| \sum_{k=1}^{m} \frac{\dot{y}_{j,k}}{|\dot{y}_{j,k}|} \dot{h}_{j,k}(\boldsymbol{\theta}) \right|^2, \tag{4}$$

where frequency response function $\dot{\mathbf{h}}_j(\boldsymbol{\theta})$ has the form

$$\dot{\mathbf{h}}_j(\boldsymbol{\theta}) = \left(\exp\left(-2\pi i f_j \tau_k\left(\boldsymbol{\theta}\right)\right), \quad k \in \overline{1,m}\right)^T, f_j \in [0, f_d/2], \tag{5}$$

$\tau_k\left(\boldsymbol{\theta}\right)$ is a certain function of time delay depending on parameter $\boldsymbol{\theta}$.

Considering the case when plane wave arrived on the seismic array, unknown parameter of model (3) $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \boldsymbol{\Theta} \subset \mathbb{R}^2$ is slowness vector and the following expressions hold

$$\operatorname{tg}\alpha = \frac{\theta_2}{\theta_1}, \quad |\boldsymbol{\theta}| = \sqrt{\theta_1^2 + \theta_2^2}, \quad \sin\beta = \frac{1}{V_p|\boldsymbol{\theta}|}, \tag{6}$$

where α is the azimuth, β is the angle of incidence, V_p is the seismic p-wave velocity near the Earth's surface. Vector $\boldsymbol{\theta}$ in expression (6) is the most convenient quantity to characterize the arrival direction of a plane wave. Value equal to $1/\boldsymbol{\theta}$ is apparent velocity vector. It follows from the equalities (6) that the estimation of $\boldsymbol{\theta}$ furnishes important information for dealing with the key problem of seismic monitoring, namely, earthquake location. Denoting horizontal coordinates of array sensor in rectangular system as (c_{kx}, c_{ky}), time delays $\tau_k\left(\boldsymbol{\theta}\right)$ of plane wave propagation from point $(0,0)$ to k-th array sensor can be calculated as dot product:

$$\tau_k\left(\boldsymbol{\theta}\right) = \theta_1 c_{kx} + \theta_2 c_{ky}, \quad k \in \overline{1,m}. \tag{7}$$

Expressions $(4, 5, 6, 7)$ allow estimating slowness vector $\boldsymbol{\theta}$ by observations $\mathbf{y}_k$ of model (1).

F-K analyses of local event from Spits array data. Below will be given numerical experiments with real data taken for different seismic arrays and corresponding to different cases of slowness vector estimation via SRP-PHAT. Recently estimator (4) was performed to simulated synthetics data for different scenarios of slowness estimation [13]. But here our goal is to test this method on real seismic array data and compare it with the existing standard method.

First, we consider the event from 6.10.2006:10.35.38 recorded by Spits array. Spits array located in the Svalbard archipelago (Fig. 1) has already proven its importance for low-magnitude seismic events study for the European Arctic region [14, 15]. Example of waveforms for P and S waves of taken event to the northwest of Svalbard are shown in Fig. 1c for several vertical components of array sensors.

An analysis of this earthquake is given in [15], where a problem arises while determining the apparent slowness vector for the S-wave. As authors noticed

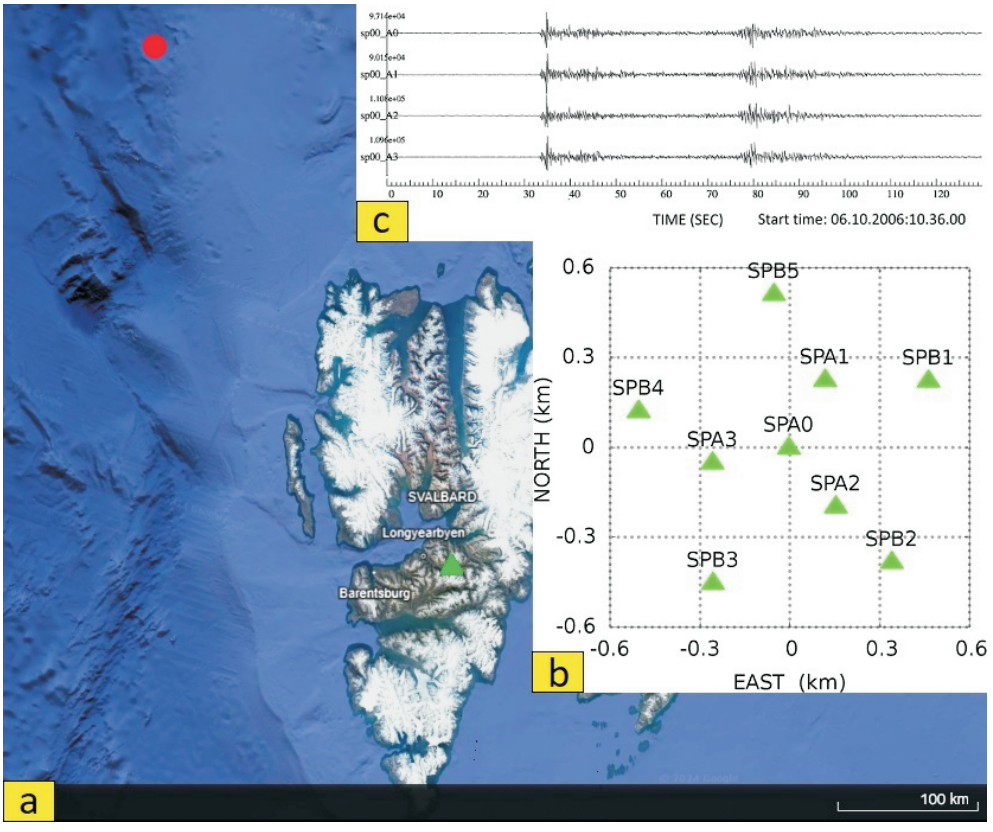


Fig. 1. a) Svalbard archipelago and relative position of Spits array and earthquake denoted by red point is at 425 km distance from Spits array; b) Spits array geometry deployed at Svalbard archipelago marked as green triangle; c) Example of 4-channel seismogram of the earthquake on 6.10.2006

that "f-k analysis on the vertical seismometers at the time of the Sn arrival is not qualitatively indicative of any coherent phase" which can be seen in Fig. 2b, while for P-waves such problem is absent which is shown in Fig. 2a. These figures were calculated by BFK method and taken from [15]. Both figures also provide supplementary information about array signal processing parameters, such as frequency band of analysis 3–5 HZ, time window of processing with length of 3.6 s, the lower and upper limits of measured slowness values from 0 to 0.4 s/km. This was useful to set up absolutely the same parameters for SRP-PHAT and compare results of slowness estimation vector for both P and S waves.

It should be noticed that the view of f-k maps 2a and 2b are not typical since that slowness system coordinates (East and North) is rotated by 180 degrees. Anyway, without loss of generality and sense of well-known definitions, in [15] authors treat the estimated azimuths 330.1 and 358.8 deg. from these maps as backazimuths.

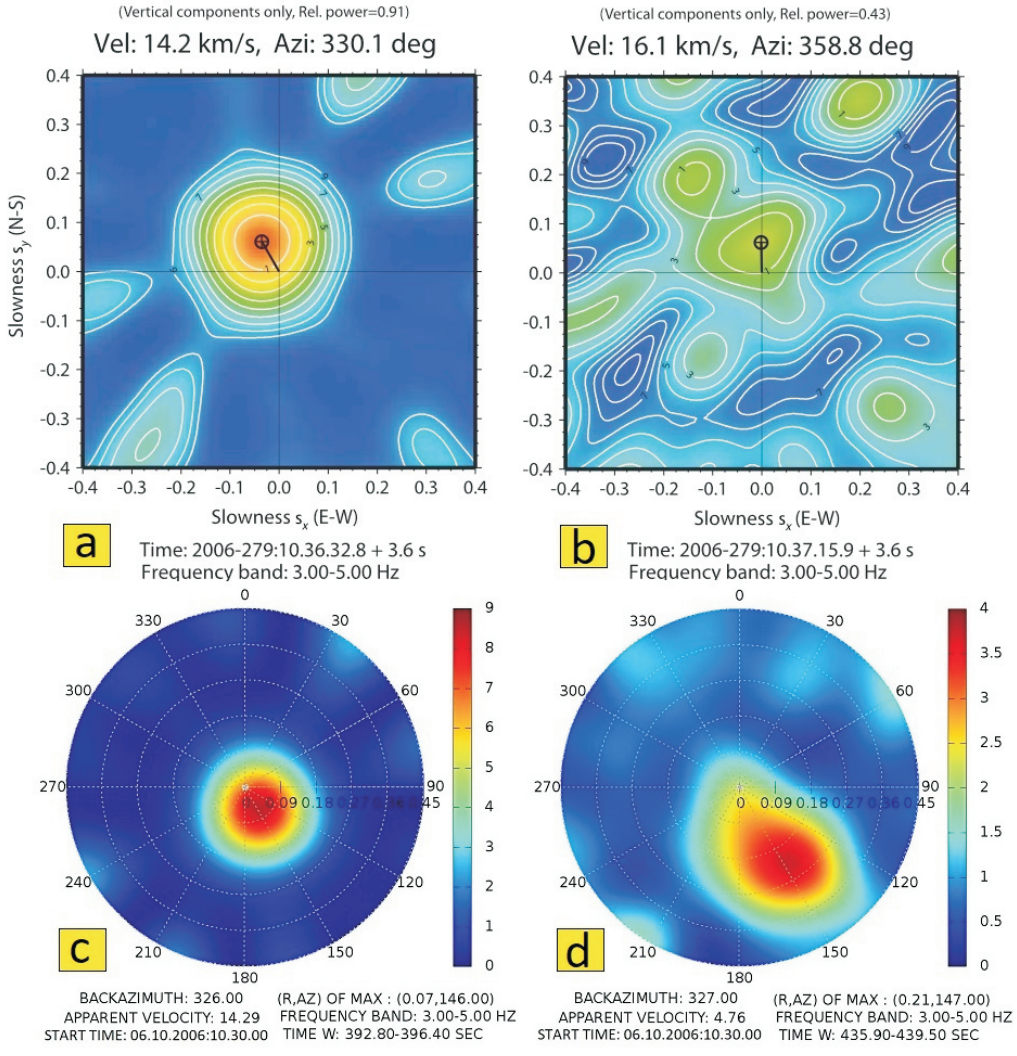


Fig. 2. f-k maps calculated by BFK – first line (a,b) and by SRP-PHAT – second line (c,d). Maps a and c were calculated for P waves and maps b and d for S waves, correspondingly

As can be seen from Fig. 2a and 2c both methods estimate slowness vector of P-wave sufficiently accurately, as evidenced firstly from the obtained values for apparent velocities: 14.2 and 14.29 km/s, secondly from the quality of f-k maps, i.e. secondary extremums of objective function are significantly less than the main one. Here we will not dwell on the discussion of the need to introduce numerical characteristics of f-k map quality, since for this study it is sufficient to do this visually. However, such characteristics exist and are considered for instance in [16] as the size or width of the peak around the main extremum in the f-k plot.

Another pair of f-k maps in Fig. 2b and 2d demonstrate significant difference

in analysis of S waves of earthquake. The estimated backazimuth value provided by SRP-PHAT for S-wave – 327 deg. stays the same as for P-wave (Fig. 2d). The estimated value of apparent velocity for S-wave is equal to 4.76 km/s that coincides with known reasonable ideas of the typical velocities for Sn seismic phases, i.e. as it follows from [17, 18] this velocity should be around 4.7 km/s in stable continental and oceanic lithosphere and less than that in more tectonically active regions. Fig. 2b has visually bad quality due to the presence of several local extremums of approximately the same value of objective function which indicates an ambiguity in determination of true value of slowness. The estimated values of backazimuth – 358.8 deg. and apparent velocity 16.1 km/s are also significantly inconsistent with physics of the process. Since the SNR value for records of P and S waves is quite high (more than $\sim$2) as seen from Fig. 1c, then problem estimation of slowness by BFK can be explained only by insufficient coherency of the observed signals between array sensors. Obviously it is true, because there is about two-fold difference between the maximum values of objective function of SRP-PHAT for the two f-k maps in Fig. 2c and 2d, so for P-wave the coherence of earthquake waveforms is better. But unlike BFK the SRP-PHAT method was able to estimate the apparent slowness for S waves accurately, which indicates its robustness.

Volcanic tremor analyses using 2 hours data of temporary 5L seismic array. Volcanic tremor is a persistent seismic signal generated by microseismicity associated with volcanic disturbances and is often associated with the movement of magmatic fluids in the subsurface. Tracking the location of its source in real time is important for reducing the volcanic risk. Using seismic array data calculation of the backazimuth and slowness in overlapping time windows helps to track the location of volcanic tremor sources. Traditional travel time location methods cannot be used to locate tremor sources, since tremor has no clear P and S wave arrivals. In contrast two or more such arrays allow a determination of the epicentral location of the source.

Temporary seismic array with network code 5L was deployed for monitoring and tracking tremor sources during subglacial floods and volcanic eruptions at Vatnajoekull ice cap in Iceland. The approximate propagation path of the volcanic dyke below the glacier is shown in Fig. 3a, and array geometry in Fig. 3b.

The volcanic tremor started on 3 September around midnight and ended abruptly at 21:35 and was studied in [6]. Two hours of the seismic array data was taken to carry out f-k analysis of the microseismicity caused by volcanic tremor Fig. 3c. We follow [6] to set up close configuration of data processing with SRP-PHAT, except for some parameters like frequency band and time window length. Spectral-temporal analysis of seismograms (Fig. 2d in [6]) shows the presence of signal mostly in the range from 0.9 to 2.0 Hz. That is why all data of seismic array was downsampled to 20 Hz. All 2 hours of multichannel seismogram is subdivided into 70% overlapping time windows that were 10 periods long – 7.8

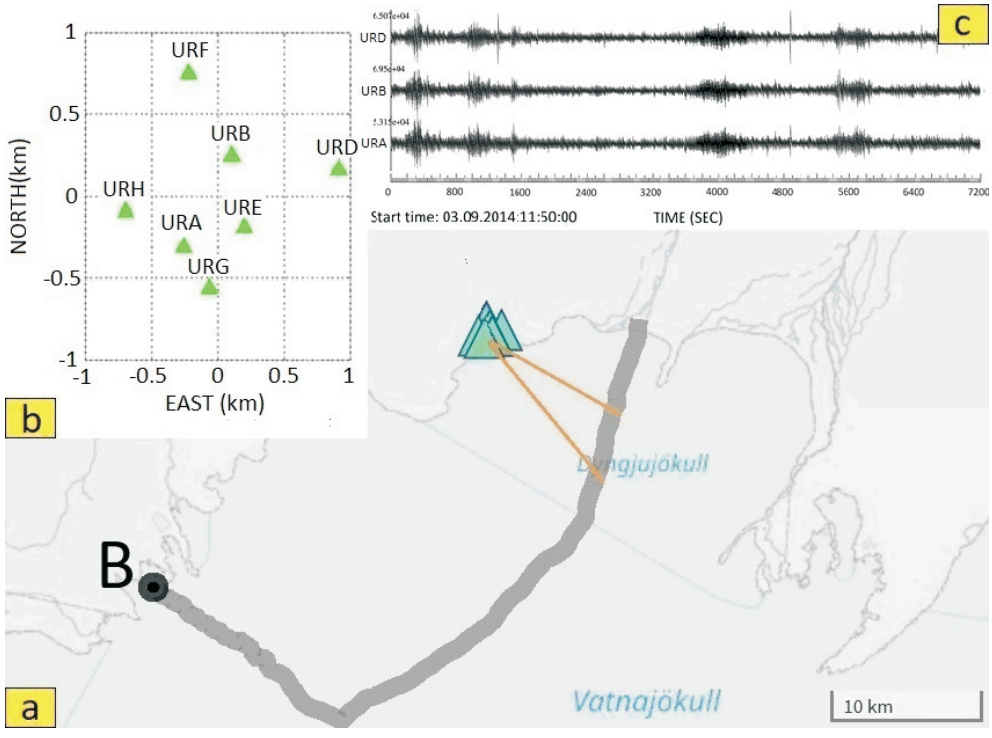


Fig. 3. a) Location of the 5L seismic array deployed to collect data during the 2014–2015 eruption of Bárðarbunga volcano, indicated by the black letter B, in Iceland. Grey line – volcanic dyke. Orange lines indicate estimated by SRP-PHAT area of backazimuth of microseismic events caused by volcanic tremor for 2 hours; b) Geometry of 5L seismic array. c) Example of 3-channel seismogram of 5L seismic array recorded on 3 September of 2014 from 11:50:00 to 13:50:00

seconds corresponding to frequency with the highest value of power spectrum ∼ 1.3 Hz.

Results of array data processing by SRP-PHAT using the settings described above are presented in Fig. 4a and b. It is clearly seen that for 2 hours from 11:50:00 to 13:50:00 volcanic tremor laterally moves in area of backazimuth from 120 to 140 degrees and vertically with respect to slowness ranged from 0.44 to 0.75 s/km. According to [6] we got a perfect match for back azimuth estimates, while slowness values are determined in a wider range – [0.44; 0.75]. Also, according to velocity model of p-waves for the region of Vatnajokull glacier (Fig. 4c), it is possible to estimate dependency between slowness values and tremor source depth. So, we can conclude that for found slownesses from 0.44 to 0.75 s/km the tremor sources are distributed at depths approximately from 0.4–0.5 to 2.3 km.

Conclusion. This paper is an attempt to bring statistically robust method SRP-PHAT in applications of seismic array data processing from related fields of knowledge. Mathematical representation of this technique justifies its applicabil-

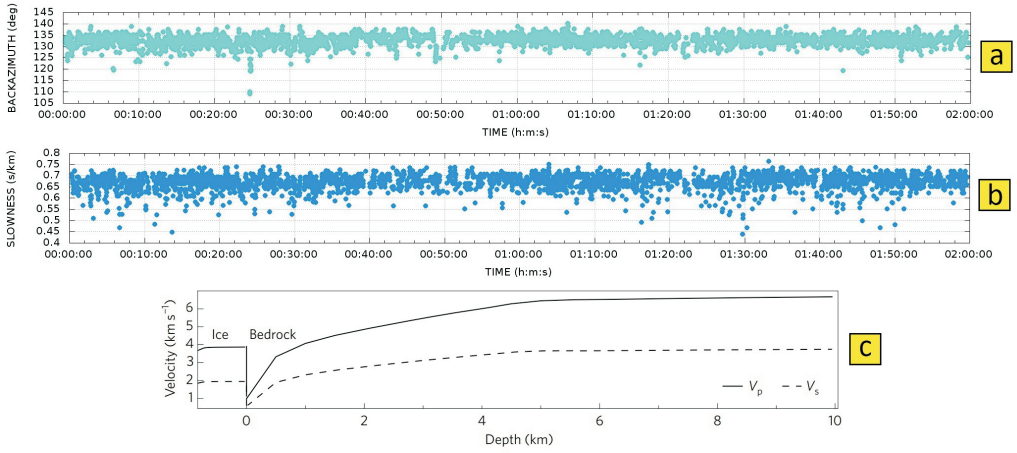


Fig. 4. Values of a) backazimuth, and b) slowness estimated in overlapping time windows of seismic array data given in Fig. 3; c) Velocity model for P and S waves in bedrock and ice in the volcanic zone around Vatnajökull

ity to f-k analyses using seismic array data. Processing of earthquake records of P and S waves of Spits array shows robustness of backazimuth and slowness estimation provided by SRP-PHAT relatively to traditional broadband f-k analyses. Solving a similar problem for 2 hours of seismic records of 5L array in overlapping time windows we managed to get results closely matching to the research in [6]. Additionally, more deep-focus volcanic tremor sources in range from 1.8 to 2.3 km were identified according to estimated slowness values, and their backazimuth ranged from 120 to 140 degrees.

For the future work, to expand the statistics of using this method, it is necessary to do more experiments with low-magnitude real seismic array data.

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