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DETERMINATION OF INTERNATIONAL SPACE STATION ORBIT CORRECTION FUEL CONSUMPTION WITH PROGRESS SHIP

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Abstract

The paper proposes a method for determining the required amount of fuel for correcting the orbit of the ISS by the spacecraft Progress and Soyuz.

Key words: ISS, correcting the orbit, spacecraft

Introduction. The altitude of the orbit of the International Space Station (ISS) is constantly changing due to the influence of solar activity and friction with the rarefied atmosphere, which leads to the gradual deceleration and loss of altitude. Atmospheric resistance lowers the altitude by an average of about 2 km per month [1]. Trajectory correction is primarily performed with the engines of the Progress and Soyuz spacecrafts [2]. Determining the amount of fuel is essential for the reliable implementation of this operation, as well as for studying the effect of the released gases on the surrounding space environment of the complex.

Transition from one trajectory to another with a change in height. The optimal trajectory of the transition between an ellipse of complementary circular orbits is a tangent semi-ellipse (Fig. 1) of the tangential maneuver, the line of which includes the radii of the circular orbits [3].

The transition based on the realization of the specified trajectory is called a multi-elliptical Hohmann transition, which for the first time showed the possibility of performing such a maneuver and predicted its optimality. The Hohmann

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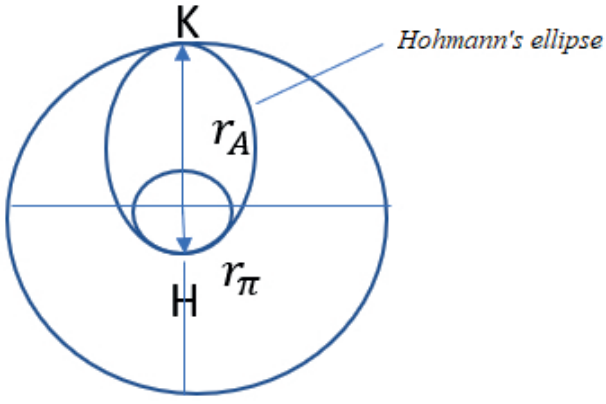


Fig. 1. Optimal transition trajectory – Hohmann's ellipse

transition trajectory is also called the minimum energy semi-ellipse. The impulse change of the speed during the execution of the maneuver takes place at the perigee and apogee of the transition orbit. The velocity V_{cr} that the spacecraft (SC) will have in the inner circular orbit can be represented as follows:

$$V_H = V_{\text{cr}} = \sqrt{\mu/r_\pi}, \tag{1}$$

where $\mu = G \cdot M$, G is the gravitational constant, and M is the object mass.

The velocity V_π required to reach the outer orbit is determined by the following formula:

$$V_\pi = \sqrt{\frac{2\mu r_A}{r_\pi(r_\pi + r_A)}}, \tag{2}$$

where r_π and r_A are the radius of the circle and the radius of the ellipse, respectively, as represented in Fig. 1.

The difference between the speeds V_π and V_H is the required speed increase ΔV_π. It is precisely such a speed that needs to be given to the apparatus in order for it to go on the new trajectory:

$$\Delta V_\pi = V_\pi - V_H = \sqrt{\mu/r_\pi}\left[\sqrt{\frac{2\mu r_A}{r_\pi(r_\pi + r_A)}} - 1\right]. \tag{3}$$

Based on the equality $r_\pi V_\pi = r_A V_A$ is obtained $V_A^2/\mu = 1/2r_\pi(r_\pi + r_A)$. To start moving the spacecraft along the outer orbit at the point K, it is necessary to apply a second velocity pulse equal to:

$$\Delta V_A = \sqrt{\frac{\mu}{r_\pi}}\left[1 - \sqrt{\frac{2\mu r_A}{r_\pi(r_\pi + r_A)}}\right]. \tag{4}$$

Using a pulse approximating action of the thrust P when turning on the propulsion system of a spacecraft having a mass m leads to a jump change in the flight speed V with a constant change in the flight speed ΔV:

$$\Delta V = \int_0^{t_k} \left(\frac{P_\tau}{m}\right) dt = J_\tau, \tag{5}$$

where the angles are, respectively:

— $\Delta\theta_V$ – orientation of the velocity vector in the plane of the initial orbit;

— $\Delta\sigma_V$ – the component of the velocity in the plane normal to the orbit.

$$\Delta\Theta_V = \frac{1}{V}\int_0^{t_k} \left(\frac{P_n}{m}\right) dt = J_n V^1; \tag{6}$$

$$\Delta\sigma_\gamma = \frac{1}{V}\int_0^{t_k} \left(\frac{P_W}{m}\right) dt = J_W V^1, \tag{7}$$

and P_τ, P_n and P_W are, respectively, the thrust vector projections of the tangent, normal, and binormal.

The action of the tangential control force of the spacecraft's motion does not change the initial position of the plane of the orbit and

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}, \tag{8}$$

where, taking into account that a is the semi-major axis of the ellipse and using the relation for the peripheral velocity $\sqrt{\mu\,(r^{-1})} = \sqrt{g_0 R^2 r^{-1}}$ the variation of the velocity V with the magnitude ΔV will cause the semi-major axis of the elliptical orbit to change:

$$(V + \Delta V) = g_0 R^2 \left(\frac{2}{r} - \frac{1}{a + \Delta a_\tau}\right). \tag{9}$$

From (9) it follows that

$$\Delta a_\tau = \frac{V_{\mathrm{I}}^2 R_r}{V_{\mathrm{I}}^2 R_r - (V - J_\tau)} - a, \tag{10}$$

where V_{I} is first space velocity.

The obtained ratios can be significantly simplified if it is taken into account that $\Delta a_\tau \ll a$ and $J_\tau = \Delta V \ll V$. Then

$$\Delta a_\tau \approx \frac{2Va^2}{V_{\mathrm{I}}^2 R} J_\tau. \tag{11}$$

The optimal trajectory of the transition between complementary circular orbits is a tangent semi-ellipse of the tangential maneuver, the line of which includes the radii of the circular orbits.

With an average deviation $r_A - r_\pi = 50\,\text{m}$ (depending on solar activity, etc.) for one day and, respectively, $\approx 1500\,\text{m}$ for 30 days; $r_A = 6789$; $r_\pi = 6786\,\text{km}$; at $R_{\text{Earth}} = 6371\,\text{km}$ and $V_{\text{I}} = 7.91\,\text{km/s}$ – first space velocity,

$$\Delta V_\pi = \left(\sqrt{\frac{2.6789}{6786+6789}} - 1\right) = 7.91\left(\sqrt{1.000221} - 1\right) = 7.91 \cdot 0.00011 \tag{12}$$
$$= 0.00087\,\text{km/s} \approx 0.87\,\text{m/s}.$$

Results and discussion. After finding ΔV_π, the amount of fuel for the specified period can be determined, assuming that the average decrease in the altitude of the ISS orbit per day is of the order of 1500 m.

The magnitude of the control impulse applied to the ISS in outer space is equal to the characteristic speed ΔV_π necessary to create the thrust P during the maneuver. In fact, the equation of motion of the spacecraft for a short flight time with the engines running has the following form:

$$m\frac{dV}{dt} = P. \tag{13}$$

The solution of this equation leads to the famous formula of K. E. Tsiolkovsky:

$$\Delta V_\pi = U_E \ln(1 + Q_\tau), \tag{14}$$

where U_E is the effective flow rate of the products of combustion from the nozzle of the engines and Q_τ is the relative fuel supply equal to the ratio of the mass of the used fuel P to the mass of the spacecraft m.

Considering that $\ln 1 = 0$ and the very small value of Q_τ ($P \ll m$), $\ln Q_\tau \approx \frac{P}{m}$, then

$$\Delta V_\pi = U_E \frac{P}{m}. \tag{15}$$

At the determined outflow velocity of the combustion products for $U_E \approx 3\,\text{km/s}$ ($\pm 15\%$), ISS mass $m \approx 400\,000\,\text{kg}$ and $\Delta V_\pi \approx 0.87\,\text{m/s}$:

$$P = \frac{400\,000 \cdot 0.87}{3000} = 116\,\text{kg}, \tag{16}$$

i.e., the fuel used for a month is approximately 116 kg or for the entire period which is 28 months 3248 kg just to correct with the freighter Progress.

The obtained results are comparable to those of the real correction on May 18, 2015 with the spaceship Progress with parameters: $m = 400\,000\,\text{kg}$, $\Delta V = 1.18\,\text{m/s}$, $\Delta t = 519.8\,\text{s}$, $\Delta H = 1.94\,\text{km}$, where for P we get

$$P = \frac{400\,000 \cdot 1.18}{3000} = 157.33\,\text{kg}.$$

Conclusion. Comparable are the amounts of fuel used, for example, by three ships: ATV Jules Verne (2008), Johann Kepler (2011) and Edoardo Amaldi (2012), which together performed 25 maneuvers to ensure station speed 67 m/s with a consumption of 8400 kg of fuel. Fuel consumption for orientation control is additionally 1926 kg [2]. Increasing the mass of the ISS to its final composition has resulted in increased correction fuel costs. In its configuration as of March 2011, the mass of the station is 375 727 kg against a planned 400 000 kg for the final configuration, and the available crew habitable space is 907 m^3. Its dimensions from the most extreme points are: 109 m wide, 51 m long and 20 m high [1].

The proposed method for determining the amount of fuel needed to correct the orbit of the ISS is practically applicable in the preliminary calculations and study of the influence of the gas jets of the engines on the environment of the station.

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