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FEATURES OF NETWORK FLOW SYSTEMS FOR DECISION MAKING WITH MOTIVATION ACCOUNTING

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Abstract

The present work investigates the problem of considering the influence of motivation in discrete network flow decision-making systems. Motivation is the primary factor that drives people to achieve their goals and plays a key role in decision-making systems. A method is proposed by which this influence is accounted for using the arc flow function amplification or attenuation coefficients. These operations are in the class of generalized network flows (flows with gains and losses). Methods to determine both the maximal and minimal value of the generalized network flow with consideration of the motivation are proposed. A numerical example of a decision-making system based on a generalized network flow with motivation accounting is described, which confirms the obtained theoretical results.

Key words: network flow systems, intelligent systems, decision-making systems, motivation

Introduction. Motivation is an important process in psychology that can significantly influence decisions in various human-machine discrete control systems. Motivation leads either to an increase in efficiency if the performers – an individual or a team – are well motivated, or to a deterioration of this efficiency – in the opposite case. The influence of motivation can be estimated fairly accurately through mathematical models that describe decision-making systems. Such are the systems for transporting (transferring) resources along individual sections

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of a network (graph) [1–3] – from one point to another. Various applications of the graph theory are known [4, 5, 8, 9].

In the present paper, it is proposed to quantify the role of motivation in decision-making systems. Furthermore, it is proposed to implement this management to use a generalized network flow, often called a flow with gains and losses [1, 2] when considering the motivation in it. In this generalized flow, the coefficients $\{g_{ij} / (i, j) \in J\}$ for strengthening or weakening of arc flow functions play an essential role.

Network flows or flows on graphs, including generalized network flows, are relatively well studied and described in many sources, including [2, 3]. In what follows, notation and terminology will be used from these sources.

The subject of human motivation occupies an increasingly significant place in the modern complex and changing social and economic environment [11, 12, 17]. Motivation is a complex set of mental processes determining the strength and direction of human behaviour [18–20]. A detailed analysis of the motivation process mechanism is presented in [13, 14]. Motivation as an essential factor in the daily need to make different kinds of decisions can be combined into a decision-making or decision-support system with motivation accounting [15, 16].

The analysis of the results shows that the network flow control system based on graph is the most appropriate way of embedding motivation [6, 7, 10]. The numerical example in Fig. 1 also describes how this can be done. In the column from Fig. 1 the number of points is 5,

$$(1) \quad X = \{x_1, x_2, x_3, x_4, x_5\}; \quad I = \{1, 2, 3, 4, 5\},$$

where X is the set of all grid points; I is the set of indices of all vertices – X .

Then $X = \{i / i \in I\}$.

In Fig. 1 the flow function f_{ij} is marked on the arc x_{ij} in brackets – the arc throughput (C_{ij}), and in a circle – the arc estimate a_{ij} .

The following notations will be introduced:

U is the set of all arcs $\{x_i, x_j\} = x_{ij}$ of the network; J is the set of the indices of all arcs of the network.

Then:

$$(2) \quad V = \{x_{ij} / (i, j) \in J\};$$

$$(3) \quad J = \{(i, j) / x_{ij} \in X\}.$$

The following arc functions will be used:

$$(4) \quad 0 \leq f_{ij} \quad \text{for each} \quad (i, j) \in J;$$

$$(5) \quad f_{ij} \leq C_{ij} \quad \text{for each} \quad (i, j) \in J;$$

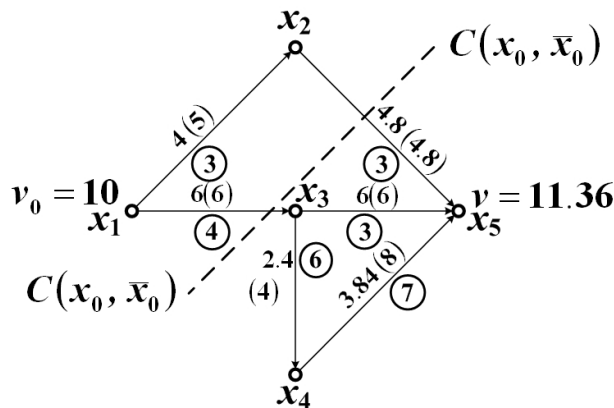


Fig. 1. Network flow control systems based on graph

$$(6) \quad 0 \leq g_{ij} \quad \text{for each} \quad (i, j) \in J,$$

where f_{ij} is the arc flow function for the arc $x_{ij} \in V$; C_{ij} is the arc throughput, which defines the upper bound of the arc flow function f_{ij} ; g_{ij} is the amplification or attenuation coefficient of the arc flow function f_{ij} .

In the generalized network model, motivation accounting can be done through the gain or loss coefficients of the arc flow functions $g_{ij} \geq 0$; $(i, j) \in J$. Conventionally, these coefficients will be called motivation coefficients.

In that:

a) If $0 \leq g_{ij} \leq 1$, the motivation decreases the value of the flow function, which already acquires the value:

$$(7) \quad g_{ij} f_{ij} < f_{ij}; \quad 0 \leq g_{ij} < 1;$$

b) If g_{ij} has the value:

$$(8) \quad g_{ij} f_{ij} > f_{ij}; \quad g_{ij} > 1,$$

then motivation enhances the effect of transferring a resource from point x_i to point x_j ;

c) If $g_{ij} = 1$, the motivation does not affect the transfer of resources, i.e.:

$$(9) \quad g_{ij} f_{ij} = f_{ij}; \quad g_{ij} = 1.$$

What value the coefficient g_{ij} will take depends on how the external and internal motivations of the decision-maker for the section (arc) x_{ij} will be evaluated by experts. If his motivation is significant, the amount of resource moved along this section will be greater and therefore $g_{ij} > 1$.

If the person interested in speeding up the process (for example, a system owner) takes the appropriate measures and increases the motivation in individual

sections, the total amount of transferred (moved/transported) resources will also be greater.

And the determination of what amounts of resources will be distributed along the individual sections (arcs) of the network is the subject of a corresponding optimization problem. To define it, it is necessary to determine the prices $\{a_{ij} / (i, j) \in J\}$ to transport a unit of resource along the section (arc) x_{ij} .

These prices – also called arc prices, always have a non-negative value:

$$(10) \quad 0 \leq a_{ij} \quad \text{for each} \quad (i, j) \in J.$$

Generalized network flow can be broadly defined by the following dependencies:

for each $i \in I$ and $(i, j) \in J$:

$$(11) \quad \sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} g_{ji} f_{ji} = \begin{cases} v_0, & \text{if } x_i = S; \\ 0, & \text{if } x_i \neq S, T; \\ -v, & \text{if } x_i = T; \end{cases}$$

$$(12) \quad f_{ij} \leq C_{ij} \quad \text{for each} \quad (i, j) \in J;$$

$$(13) \quad 0 \leq f_{ij} \quad \text{for each} \quad (i, j) \in J.$$

On the generalized network flow with motivations defined in this way, at least two optimization problems can be formulated – A and B with different goal functions:

Problem A: Maximal generalized network flow with motivations and the following goal function:

$$(14) \quad L = v = v_{\max} \rightarrow \max$$

where v_0 is the amount of initial resource at the vertex $S = x_1$ of the network.

Problem B: Maximal generalized network flow with motivations and with the minimal or maximal value of the following goal function:

$$(15) \quad L = \sum_{(i,j) \in J} a_{ij} f_{ij} \rightarrow \min (\max),$$

where v_0 is the amount of initial resource $v = v_{\max}$ and was obtained when solving problem A .

In the considered example, the coefficients $\{g_{ij} / (i, j)\}$ have the following values:

$$(16) \quad g_{1,2} = 1.2; \quad g_{1,3} = 1.4; \quad g_{2,5} = 0.8; \quad g_{3,4} = 1.6; \quad g_{3,5} = 1.1; \quad g_{4,5} = 0.5.$$

Arc throughputs are equal to:

$$(17) \quad C_{1,2} = 5; \quad C_{1,3} = 6; \quad C_{2,5} = 4.8; \quad C_{3,4} = 4; \quad C_{3,5} = 6; \quad C_{4,5} = 8.$$

The given data makes it possible to solve each of the described optimization problems A and B for a generalized network flow with consideration of the motivations.

Optimization problem A. In this maximal generalized network flow problem with motivations, the goal function is defined by (14) and the constraints (11) have the following form:

$$(18) \quad a_1 : f_{1,2} + f_{1,3} = 10;$$

$$a_2 : f_{2,5} - 1, 2f_{1,2} = 0;$$

$$a_3 : f_{3,4} + f_{3,5} - 1, 4f_{1,3} = 0;$$

$$a_4 : f_{4,5} - 1, 6f_{3,4} = 0;$$

$$(19) \quad a_5 : -0, 8f_{2,5} - 1, 1f_{3,5} - 0, 5f_{4,5} + v = 0.$$

The constraints from (12) and (13) are described by the dependencies:

$$a_6 : f_{1,2} \leq 5;$$

$$a_7 : f_{1,3} \leq 6;$$

$$a_8 : f_{2,5} \leq 4.8;$$

$$a_9 : f_{3,4} \leq 4;$$

$$a_{10} : f_{3,5} \leq 6;$$

$$a_{11} : f_{4,5} \leq 8;$$

$$a_{12} : f_{1,2} \geq 5;$$

$$a_{13} : f_{1,3} \geq 0;$$

$$a_{14} : f_{2,5} \geq 0;$$

$$a_{15} : f_{3,4} \geq 0;$$

$$a_{16} : f_{3,5} \geq 0;$$

$$a_{17} : f_{4,5} \geq 0.$$

In the considered numerical example, the constraints consist of a total of 17 equalities and inequalities.

Solving the above maxflow optimization problem using the standard linear programming package LPSolveIDE shows that the maximal possible flow of resources from the source x_1 to the consumer x_5 is equal to $v = 11.36$. This means

that through the use of the $\{g_{ij} / (i, j) \in J\}$ motivation, the resource transported from the source to the consumer will increase by 11.36%.

The coefficients $\{g_{ij} / (i, j) \in J\}$ of the arcs entering the vertices $\{x_2, x_3, x_4\}$, thanks to the motivation take an additional resource from these three vertices to increase the final transported resource from $v_0 = 10$ to $v = 11.36\%$.

This clearly shows how motivation can influence decision-making and the increase (or decrease) of the transport resource.

In the next optimization problem B under consideration, the arc estimates (10) have the following values:

$$(20) \quad a_{1,2} = 3; \quad a_{1,3} = 4; \quad a_{2,5} = 3; \quad a_{3,4} = 6; \quad a_{3,5} = 3; \quad a_{4,5} = 7.$$

Optimization problem B . In this problem, the maximal generalized network flow of motivations whose goal function L is defined in (15) is determined. This is the so-called main cost-maxflow problem. The first step in it is solving the optimization problem A to determine the maximal generalized network flow with motivations, in which it is assumed that $v_0 = 10$. After solving this problem is obtained $v = v_{\max} = 11.36\%$.

After the determination of $v = v_{\max}$, we proceed to the second step where the equation a_5 takes the form:

$$(21) \quad a_5 = -0.8f_{2,5} - 1.1f_{3,5} - 0.5f_{4,5} = -11.36$$

and the goal function L is equal to:

$$(22) \quad L = 3f_{1,2} + 4f_{1,3} + 3f_{2,5} + 6f_{3,4} + 3f_{3,5} + 7f_{4,5} \rightarrow \min.$$

A new optimization problem is solved with the constraints from a_1 to a_{17} where a_5 from (21) is used instead of a_5 , and the goal function L is equal to (22). The LPSolveIDE linear programming package used leads to the following arc flow functions:

$$(23) \quad f_{1,2} = 4; \quad f_{1,3} = 6; \quad f_{2,5} = 4.8; \quad f_{3,4} = 2.4; \quad f_{3,5} = 6; \quad f_{4,5} = 3.84.$$

This is the optimal distribution of the transported resource in a network along the individual sections (arcs) of the network and taking into account the influence of motivation, it has the same general effect of increasing the quantities and decreasing the total cost of transporting resources. And in the optimization problem B – as well as in problem A , an increase in the transported resource by 11.36% is obtained.

The total value of the transported maximal possible resource with a minimal value is:

$$(24) \quad L = \sum_{(i,j) \in J} a_{ij} f_{ij} = 3 \cdot 4 + 4 \cdot 6 + 3 \cdot 4.8 + 6 \cdot 2.4 + 3 \cdot 6 + 7 \cdot 3.84 = 109.68 \text{ units.}$$

The maximal possible flow (resource) from the point x_1 in quantity 10 cannot be transported, taking into account the motivations, to a point x_5 for a value less than 109.68 units. At the same time, the amount of transported resource is $v = v_{\max} = 11.36$.

From the obtained solutions it follows that the arcs $\{x_{1,3}, x_{2,5}\}$ are saturated, i.e.:

$$(25) \quad f_{1,3} = C_{1,3} = 6 \quad \text{and} \quad f_{2,5} = C_{2,5} = 4.8.$$

They form a cut:

$$(26) \quad (X_0, \bar{X}_0) = \{x_{1,3}; x_{2,5}\}; \quad (\bar{X}_0, X_0) = \emptyset,$$

where $\emptyset$ is the empty set.

Conclusion. The results obtained in the present paper provide an opportunity to account for motivation in a discrete decision-making system through a generalized network flow. The usefulness of such an approach is clearly shown. It makes it possible to bridge the gap between psychological processes, motivation in particular, and rigorous models of discrete decision-making systems.

The obtained positive results make it possible to reveal new directions for research and to create new decision-making systems with consideration of other psychological processes.

It is of interest if in the described class of discrete decision-making systems through a generalized network flow the behaviour of such systems is examined if the motivation changes dynamically and has a stochastic character.

There is a need for the combination of psychological and mathematical methods, which will create greater efficiency of the combined methods of taking solutions compared to using them separately. And this, in turn, leads to the emergence of new research tasks in decision-making processes.

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