Abstract

Elevated storage tanks are used to store fresh and waste water, cereal, oil and petrochemical materials. Therefore they serve in a wide area of utilization in this day and age. Analysis of a half full with water elevated storage tank is performed in the scope of this study. The structure is composed of a steel circular storage tank and frame typed steel stager. The structure is under the effect of earthquake loads as well as operational ones. Numerical model is generated by Finite Elements Analysis (FEA) software including fluid and structure parts. The interaction of the parts is provided by Coupled Eulerian Lagrangian (CEL) technique. While the structure constitutes the Lagrangian part, the fluid constitutes the Eulerian part. Interaction of Eulerian and Lagrangian parts are provided by general contact algorithms. Free surface movement of the water, maximum displacements and natural frequency values with related mode shapes of the structure are obtained after numerical analysis. Numerical results are verified by semi-analytical model, where the structure is modelled as single degree of freedom system. The accordance between results is numerically and visually obtained. The turbulent movement of water under the influence of an earthquake is modelled by utilizing the CEL technique. So, the effect of the dynamic pressure induced by the turbulence on the tank and the structural system has been taken into consideration.

**Key words:** elevated storage tank, fluid structure interaction, numerical analysis, coupled Eulerian Lagrangian technique

https://doi.org/10.7546/CRABS.2024.05.10
**Introduction.** Storage tanks are important structures installed underground, above ground or elevated from the ground, manufactured from mainly stainless steel or concrete. They are important structures because they are used to store all kinds of important materials from water to nuclear materials. During their service life, storage tanks are faced with environmental loads such as especially earthquakes and wind, as well as damage threats that may occur due to blast and fire [1].

Analysis of fluid storage tanks is a vexed issue by comparison with other type of storage tanks due to complexity and difficulty of fluid structure interaction. Elevated fluid storage tanks are the most fragile ones under seismic loads among anchored, unanchored and elevated ones due to bearing heavy water body on their tall and slender staging system [2,3]. External forces particularly earthquakes induce large structure deformations and sloshing on the free surface. Sloshing is destructive when the predominant forcing frequency of the earthquake is near the sloshing frequency considering the resonance phenomenon [4]. Even so only few of the studies in the literature practise fluid structure interaction including free surface modelling. In the study of KAMARROUDI et al. [5] shaking table experiments were performed to obtain maximum sloshing height of water in the elevated concrete tank. The result is verified by Abaqus FEA software. The verified numeric model is used to simulate elevated tanks under seismic loads. While sloshing response of water is performed according to dynamic equilibrium equation, time history of horizontal displacements of the elevated steel tank is numerically performed under both single and multiple earthquakes in [6]. GHOOHESTANI et al. [7] used FEA to compute the vibration equation of steel tank elevated by concrete shaft. Velocity potential theory is used to achieve sloshing height in this study. PATHAK and MISHRA [8] have evaluated seismic behaviour of elevated storage tank, with different vertical bracing systems in staging. Two degree of freedom model for different staging patterns are developed to achieve structural outputs and sloshing heights.

In the view of the researches in literature, several methods have been utilized to investigate the dynamic behaviour of the elevated fluid storage tanks. Different domains of physical problem such as structures and fluids can be modelled simultaneously using Lagrangian and Eulerian approaches in FEA based numerical techniques [9]. Arbitrary Lagrangian Eulerian (ALE) and Coupled Eulerian Lagrangian (CEL) procedures where free surface motions can be modelled are used in FEA based modelling. CEL method is performed by [10] to model the behaviour of concrete elevated tank under impact load. Sloshing height of water is achieved for different water levels in the tank. Thus water level effects on the stiffness of the tank are observed. In the study of BAGHBAN et al. [11] ALE method is performed to determine the effect of baffles on the sloshing of fluid surface in the steel-elevated storage tank. The studies of [12,13] can be given as an example for performing CEL and ALE methods in sloshing modelling of different
types of storage tanks. Abaqus software is widely used to model the ALE and CEL aided interaction [14,15].

The origin point of this study is the limited number of studies in which the free surface movement of the water in the elevated tank under seismic load is modelled and the bidirectional fluid structure interaction is carried out. This paper aims to numerically investigate the dynamic behaviour of elevated tank and its staging impressed by seismic load. CEL technique is utilized by computer software in the analysis. The numeric model is verified by semi-analytical method in which the structure is defined as a two degree of freedom system by lumped masses [16]. Verification is performed via natural frequency and top displacement of the structure. At the same time, structural parameters such as reaction forces in the staging, stress distribution of staging and the tank, movement of the water in the tank are achieved in the end.

Description of the structure. The numerical model of elevated storage tank and its staging are seen in Fig 1a. Cylindrical tank is mounted on the six legged jacket type staging. Legs of the staging are connected by diagonal members. Diameter \((D)\), height \((h_t)\) and the thickness \((s)\) of the tank are 5.0 m, 5.0 m and 0.015 m, respectively. While the legs of the staging are box sections with dimensions of 0.25 m \(\times\) 0.25 m and thickness of 0.01 m, diagonals of it are same section type and same thickness value with dimensions of 0.12 m \(\times\) 0.12 m. Since the staging height \((h_s)\) is 6.0 m, total height \((H)\) of the model is 11.0 m. Steel material with a Young’s modulus of 210 GPa, Poisson’s ratio of 0.3 and the density value of 7850 kg/m\(^3\) is used in tank and staging. Water occupies half volume of the tank and it is modelled as EOS material with the velocity of sound \((c_0)\) in water 1450 m/s, with the density \((\rho)\) of 1025 kg/m\(^3\) and the dynamic viscosity \((\mu)\) of 0.0015 Ns/m\(^2\).

Analyses. The analyses are carried out for two main parts such as Coupled Eulerian Lagrangian (CEL) and semi-analytical analyses. While the geometric structure of the model used in CEL analysis is shown in Fig 1a, the mesh structure
is seen in Fig 1b. Simplified version of the same model utilized in semi-analytical analysis is shown in Fig 1c. In addition, the seismic acceleration record effecting the structure used in both analyses is given in Fig 1d.

**CEL analysis.** In the numerical analysis, Abaqus utilizes the combination of Eulerian–Lagrangian approach defined as (CEL) procedure by the equations listed below. Eqs. (1–3) are mass, momentum and conservation of energy equations in turn.

\[
\begin{align*}
(1) & \quad \frac{D\rho}{Dt} + \rho \nabla \cdot v = 0 \\
(2) & \quad \rho \frac{Dv}{Dt} = \nabla \cdot \sigma + \rho b \\
(3) & \quad \frac{De}{Dt} = \sigma : D.
\end{align*}
\]

In the equations, \(v\) is material velocity, \(\rho\) is density, \(\sigma\) is the Cauchy stress, \(b\) is the body force, and \(e\) is the internal energy per unit volume.

\[
(4) \quad \frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + v \cdot (\nabla \varphi).
\]

Conservation equations used for Lagrangian approach are obtained in general form of Eulerian approach by using Eq. (4). However, \(\varphi\) is randomly selected solution factor.

\[
(5) \quad \frac{\partial \varphi}{\partial t} + \nabla \cdot \Phi = S.
\]

In Eq. (5), \(\Phi\) defines the flux function and \(S\) symbolizes the source term. The equation may also be shown as two independent equations as follows:

\[
(6) \quad \frac{\partial \varphi}{\partial t} = S
\]

\[
(7) \quad \frac{\partial \varphi}{\partial t} + \nabla \cdot \Phi = 0.
\]

As the spatial time derivative is changed by the material time derivative on the fixed mesh, Eq. (6) turns into same with the standard Lagrangian formulation. The deformed mesh is transferred to the original fixed mesh, and the volume of material transported between adjacent elements is needed to be computed for solving the Eq. (7). Mass, energy, momentum and stress parameters for the Lagrangian procedure are expressed for the flow of the material between adjacent elements via transport algorithms.
**CEL application to template structure.** In this study, bidirectional fluid structure interaction analysis is generated by applying CEL technique. Eulerian section is comprised of water assigned and unassigned (void) sections as it is seen in Fig 1a. Boundary conditions as fixed supports and mesh structure of the numerical model are presented in Fig. 1b. Material characteristics are defined for the specific parts. After completing the assign of the material properties, seismic acceleration record in Fig 1d is applied to the structure. While S3 elements are used in Lagrangian part, EC3D8R elements are utilized in Eulerian part. The distance between nodes in Lagrangian section is 0.015 m with the same value of the thickness. The same value is used in Eulerian part. As a result, the entire finite elements model is constituted by 779 444 nodes and 788 055 elements on staging, 2 353 932 nodes and 4 707 856 elements on tank and 994 900 nodes 969 408 elements on water geometries.

The equation of motion under external forces ($F$) that is used by the software can be given as follows:

$$m^{NJ} \ddot{X}^N \bigg|_t = (F^J - I^J) \bigg|_t.$$  \hspace{1cm} (8)

In Eq. (8) $m^{NJ}$ is the mass matrix, $F^J$ represents the external applied load vector transferred from the Eulerian section, $I^J$ symbolizes the internal force vector induced by internal stresses of elements, and $\ddot{X}$ is the acceleration. $I^J$ is acquired from the independent elements so that a global stiffness matrix which is not necessary to be generated. Coupled Eulerian–Lagrangian analysis could be carried out in dynamic, explicit steps only [14]. Explicit integration rule presented by the following equations is performed to determine the displacements which are carried over to fluid from the structure.

$$\dot{X}^{N}_{(i+\frac{1}{2})} = \dot{X}^{N}_{(i-\frac{1}{2})} + \frac{\Delta t_{(i+1)} + \Delta t_{(i)}}{2} \ddot{X}^{N}_{(i)}.$$  \hspace{1cm} (9)

$$X^{N}_{(i+1)} = X^{N}_{(i)} + \Delta t_{(t+1)} \dot{X}^{N}_{(i+\frac{1}{2})}.$$  \hspace{1cm} (10)

$X^N$ is a degree of freedom of displacement component and the subscript $i$ is the increment number of explicit dynamics step. The central-difference integration operator is explicit with regards to the advancement of the kinematic state due to the known values of $\dot{X}^{N}_{(i-\frac{1}{2})}$ and $\ddot{X}^{N}_{(i)}$ taken from the previous increment. The nodal accelerations are determined by Eq. (11) as follows:

$$\ddot{X}^{N}_{(i)} = (m^{NJ})^{-1} (P^J - I^J).$$  \hspace{1cm} (11)

Iterations are not necessary in the referent procedure to revise the values of the displacement, velocity and acceleration. Modal analyses are also performed.
simultaneously to obtain natural frequencies in addition to explicit analysis. The matrices are used to define the finite element of the model in Eq. (12). Lanczos procedure is used to solve the matrices [14] in which $\lambda$ defines the square of natural frequency.

\begin{equation}
[k]{X} - \lambda [m] {X} = 0.
\end{equation}

**Semi-analytical analysis application to template structure.** In this part of the study the structure is idealized as a lumped mass tower as it is seen in Fig 1d. Analysis of two degree of freedom system under seismic load is utilized by Eq. (14) via application of the coordinate transformation given by Eq. (13). Modal shape matrix $[\phi]$ acquired due to the structural modes is engaged in the transformation of coordinates.

\begin{equation}
\{X\} = [\phi] \{\xi\}
\end{equation}

\begin{equation}
\end{equation}

The equation is solved by the beginning conditions by Runge–Kutta procedure to determine the point displacements.

\begin{equation}
\xi_{1(0)} = \xi_{2(0)} = 0, \quad \dot{\xi}_{1(0)} = \dot{\xi}_{2(0)} = 0.
\end{equation}

The Runge–Kutta method evaluates the simple relationships at the beginning, middle and end of all overall time steps ($\Delta t$) [17].

\begin{equation}
\ddot{X}(t) = m^{-1}(F(t) - kX(t)) \quad \ddot{X}_{t+\Delta t} = \dot{X}_t + \dddot{X}_t \Delta t \quad X_{t+\Delta t} = X_t + \dot{X}_t \Delta t.
\end{equation}

Aside from the displacements, natural frequency of the empty structure ($\omega$) is determined by Eq. (17).

\begin{equation}
[k] - \omega^2 [m] = 0.
\end{equation}

Matrices of mass and stiffness that are computed according to rules given by [18,19] are presented by the following equations:

\begin{equation}
k = \begin{bmatrix} 41.09 \times 10^4 & -10.09 x \times 10^4 \\ -10.09 x \times 10^4 & 10.09 x \times 10^4 \end{bmatrix}.
\end{equation}

\begin{equation}
m = \begin{bmatrix} 42.84 \times 10^3 & 0 \\ 0 & 14.59 \times 10^3 \end{bmatrix}.
\end{equation}

Furthermore, modal shape matrix is turned into the matrix form as follows:

\begin{equation}
\phi = \begin{bmatrix} 1 & 1 \\ 1.45 & -0.12 \end{bmatrix}.
\end{equation}
Multiplication of mass matrix and seismic acceleration record constitutes the external forces of right side in equation of motion. Eqs. (18–20) are placed into the equation of motion in Eq. (14) and the equation is solved. Both semi-analytical and finite elements methods are continued for 60 s with the length of the step interval ($\Delta t$) 0.01 s.

Results. In this section, the numeric and semi-analytical outputs of the structure are comparatively evaluated. Thus, verification of the numeric models is provided via displacement and natural frequency values. First three natural frequencies and corresponding mode shapes of the empty structure are given in Fig 2.

![Fig. 2. Mode shapes and natural frequency values](image)

Natural frequency values which are determined from the software are obtained by Eq. (17) as well. While first natural frequency of the two degree of freedom system is computed as 7.5717 rad/s, second natural frequency value is obtained as 7.6861 rad/s. Distributions of the structural outputs as displacement, Von Mises

![Fig. 3. Distributions of the structural outputs. a. Displacement b. Von Mises stress c. Reaction force](image)
stress and reaction force are given in Fig 3. While the displacement value at the top point of the model is 0.1812 m in the numerical analysis, it is calculated as 0.1954 m in the semi-analytical analysis. The values of maximum stress and reaction force in the structural system are $8.882 \times 10^8$ N/m$^2$ and $1.910 \times 10^6$ N, respectively. On the other hand, it is seen that the maximum stress in the tank has occurred on the surfaces in contact with water.

In the final step of the results section, coupling of Eulerian and Lagrangian sections is given in Fig 4. Oscillation of the water in the structure is also obtained as well as structural results. Structural displacement changing owing to the wave movement and the free surface elevations of the wave for various time intervals are seen in Fig 4.

![Fig. 4. Oscillation of the water in the structure](image)

**Conclusions.** In this study, fluid-structure interaction analysis of elevated storage tank under seismic record is performed according to two different analyses. While the first analysis type is numerical, the other one is semi-analytical. Bidirectional fluid-structure interaction analysis is carried out in computer environment. On the other side, unidirectional fluid-structure interaction is utilized in semi-analytical analysis. CEL approach is utilized by Abaqus software in the numerical analysis. Investigation of the numerical analysis is performed by considering structural and environmental outputs.

Oscillation of the water due to seismic load is transferred to the structure by CEL approach to finalize the structural analysis. The accuracy of the results between semi-analytical and numerical analysis are obtained. Since, number of nodes and elements establish FEM analysis more complicated owing to the increase in the height of the structure, semi-analytical procedure could be utilized as an alternative way as fluid outputs are not essential apart from structural ones.

CEL technique of this study enables performing fluid-structure interaction by single interface. In the event of resolving the same problem by only Lagrangian procedure in the place of Eulerian–Lagrangian procedures, two separate interfaces are involved as well. While one of them appertains to fluid model, the other interface is for the structural one. Interaction of the surfaces could be procured by contact surfaces. The surfaces of the solid and fluid models shall be accurately determined to ensure the interaction. As a result, mesh densification shall be per-
formed for specific zones. However, the number of nodes and elements increase in this case. Co-simulation boundary conditions are employed to define the contact surfaces in the software. Thus, forces from fluid to structure and structural displacements to fluid are transferred individually. One single interface is utilized to define the contact surfaces and the interaction in CEL analyses. Penalty contact in the software is used to provide the contact between surfaces.

High capacity computer is needed as a result of the utilization of various interfaces and increase of node and element numbers. Otherwise, solution time extends far too much. Therefore, both Eulerian–Lagrangian procedures are used for similar fluid-structure interaction problems to provide advantage. Besides, it is also known that utilizing Eulerian–Lagrangian methods together yields more accurate results for the interaction problems under the effect of major deformations.

In this study, the behaviour of a single liquid in the structure and its effect on the structure are modelled for a single filling rate. The behaviour of the structure for various filling rates and the movement of different liquids under seismic effect will be investigated in future studies. Also, it is considered to use different stagers and combination types in the structural analyses.

REFERENCES


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