

STATE OF CHARGE ESTIMATION FOR LITHIUM
BATTERY BASED ON FRACTIONAL ORDER SQUARE
ROOT CUBATURE KALMAN FILTER AND ADAPTIVE
MULTI-INNOVATION UNSCENTED KALMAN FILTER

Ying Wei✉

Received on January 26, 2024

Presented by S. Margenov, Corresponding Member of BAS, on February 27, 2024

Abstract

Accurate state of charge (SOC) estimation of batteries is of great significance for electric vehicles. A SOC estimation method based on a fractional order square root cubature Kalman filter (FOSRCKF) and an adaptive multi-innovation unscented Kalman filter (AMIUKF) is proposed. The battery is modelled using fractional order calculus theory and the model parameters are identified by adaptive genetic algorithm. The FOSRCKF estimates the battery SOC, while the AMIUKF online updates the internal resistance in the model, and there exchanges information between two filters. The experimental results under the Urban Dynamometer Driving Schedule (UDDS) and the US06 Highway Driving Schedule show that the proposed method has lower SOC estimation error and lower terminal prediction error compared with the traditional SRCKF method based on integer order models, which demonstrates the effectiveness, accuracy and robustness of the proposed method.

Key words: SOC estimation, fractional order model, square root cubature Kalman filter, adaptive multi-innovation unscented Kalman filter

2020 Mathematics Subject Classification: 62H12, 93C35, 94C60

This research was funded by the project of the Natural Science Foundation of the Higher Education Institute of Anhui Province, Grant/Award No. 2022AH051982 and the Project for Cultivating Disciplines (Majors) Leader of Colleges and Universities in Anhui Province, Grant/Award No. DTR2023062.

<https://doi.org/10.7546/CRABS.2024.04.02>

Introduction. Electric vehicles (EVs) are becoming increasingly popular due to the depletion of fossil fuels and the demand for clean air. As the power source of EVs, the safety of lithium batteries is crucial for safe driving. Battery management system (BMS) is often used for monitoring various states of batteries for example state of charge (SOC), state of health, state of energy [1]. The SOC is an indicator of the mileage, similar to the fuel gauge of a fuel vehicle, therefore the SOC estimation is the most important function of BMS.

The SOC of batteries cannot be measured and only can be estimated through measuring the battery current, terminal voltage or temperatures. At present, the existing SOC estimation methods can be roughly divided into model-free and model-based methods. Model-free methods mainly include ampere hour integration (AHI) method, open circuit voltage (OCV) method and data-driven method. The AHI method is simple and easy to implement because it calculates SOC by integrating the battery current over time. However, the AHI method does not use any feedback information to correct the estimation results of SOC, and it cannot eliminate the cumulative error caused by incorrect initial SOC value or noise in the measured current. The OCV method uses the mapping between the battery OCV and SOC to determine the SOC, which is disabled to estimate the SOC in real time in view of the fact that only by allowing the battery to stand for a long time can the open circuit voltage be effectively measured. The data-driven method considers the battery as a “black” box, which input is usually the battery current, voltage as well as temperature and the output is the estimated SOC. This method uses machine learning algorithms combined with training datasets to determine the quantitative relationship between the input (i.e. the battery current, voltage and temperature) and output (i.e. the battery SOC) without the knowledge of battery model. However, complexity of influencing factors makes it challenging to accurately get the quantitative relationship that can characterize the internal behaviour of the battery, which results in the uncertainty of the SOC estimation caused by the quantity and quality of the training datasets. In contrast, model-based SOC estimation methods have some characteristics for example self-correction and online implementations. The model-based methods estimate the SOC often by combining filtering algorithms with battery models. XIONG et al. [2] estimated the battery SOC by using extended Kalman filter (EKF) with a thermal-dependent electrical model. CHENG et al. [3] proposed a dual fuzzy-based adaptive extended Kalman filter (DFAEKF) method using a second-order RC equivalent circuit model for the SOC estimation of liquid metal batteries. Compared with EKF, unscented Kalman filter (UKF) has better accuracy and stability because UKF uses an unscented transformation to approximate the noise distribution instead of first-order Taylor series expansion. HOU et al. [4] used an enhanced adaptive UKF to simultaneously estimate the battery SOC and model parameters. REZAEI et al. [5] proposed a lithium battery SOC estimator based on a fuzzy system and UKF. In the aforementioned literatures, the most com-

monly used battery models are equivalent circuit models (ECMs) which consist of several capacitors and resistors. In these ECMs, the relationship between the voltage across the capacitors and the current follows a simple differential or integral expression, therefore we call this kind of ECMs integer order models (IOMs). For model-based methods, the accuracy of battery modelling influences that of SOC estimation. However, IOMs are not sufficient to simulate highly nonlinear behaviour inside the battery. In recent years, fractional order models (FOMs) have been applied to model-based methods for the SOC estimation. ZHU et al. [6] established a fractional order 2-RC circuit model and used adaptive EKF to accomplish the SOC estimation of lithium batteries. The work of XIONG et al. [7] demonstrated that FOMs have higher estimation accuracy in SOC and battery terminal voltage than IOMs even if the battery operated in ageing level and over wide range of temperature.

In this paper, a novel fractional order model-based method for the SOC estimation is proposed. Two filters are designed in the proposed method. The first filter is fractional order square root cubature Kalman filter (FOSRCKF) and it is used for estimating the battery SOC. The FOSRCKF not only utilizes the advantages of FOMs, but also avoids the divergence of the filter caused by the non-positive definite of the covariance matrix. Furthermore, it can be seen from the circuit model that the internal resistance (R_0) directly affects the terminal voltage of the battery and thus the SOC by multiplying it with the current. Therefore, real-time and accurate estimation of R_0 can effectively reduce the estimation error of the terminal voltage, thereby improving the accuracy of the FOSRCKF for SOC estimation. The second filter is adaptive multi-innovation unscented Kalman filter (AMIUKF) used for estimating R_0 . The effectiveness and robustness of the proposed method are finally verified under the Urban Dynamometer Driving Schedule (UDDS) and the US06 Highway Driving Schedule.

Fractional order battery modelling. A fractional order 2-RC circuit is used to model the battery. The model is constructed as shown in the upper left corner of Fig. 1.

In Figure 1, R_0 is the internal resistance, C_1 , C_2 and C_3 are the constant phase elements CPE1, CPE2 and CPE3 instead of general capacitors, both R_1 and R_2 are the resistances of RC networks, U_T and I_L are terminal voltage and load current, and U_{OC} denotes the OCV.

Fundamentals of fractional order calculus. In this study, Grünwald–Letnikov (GL) definition [8] is applied to establish fractional order equations for describing the relationship between the battery voltage and current. The GL fractional-order derivative of non-integer order α is defined as

$$(1) \quad \mathfrak{D}^\alpha x(t) = \lim_{T_s \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\lceil t/T_s \rceil} (-1)^j \binom{\alpha}{j} x(t - jT_s),$$

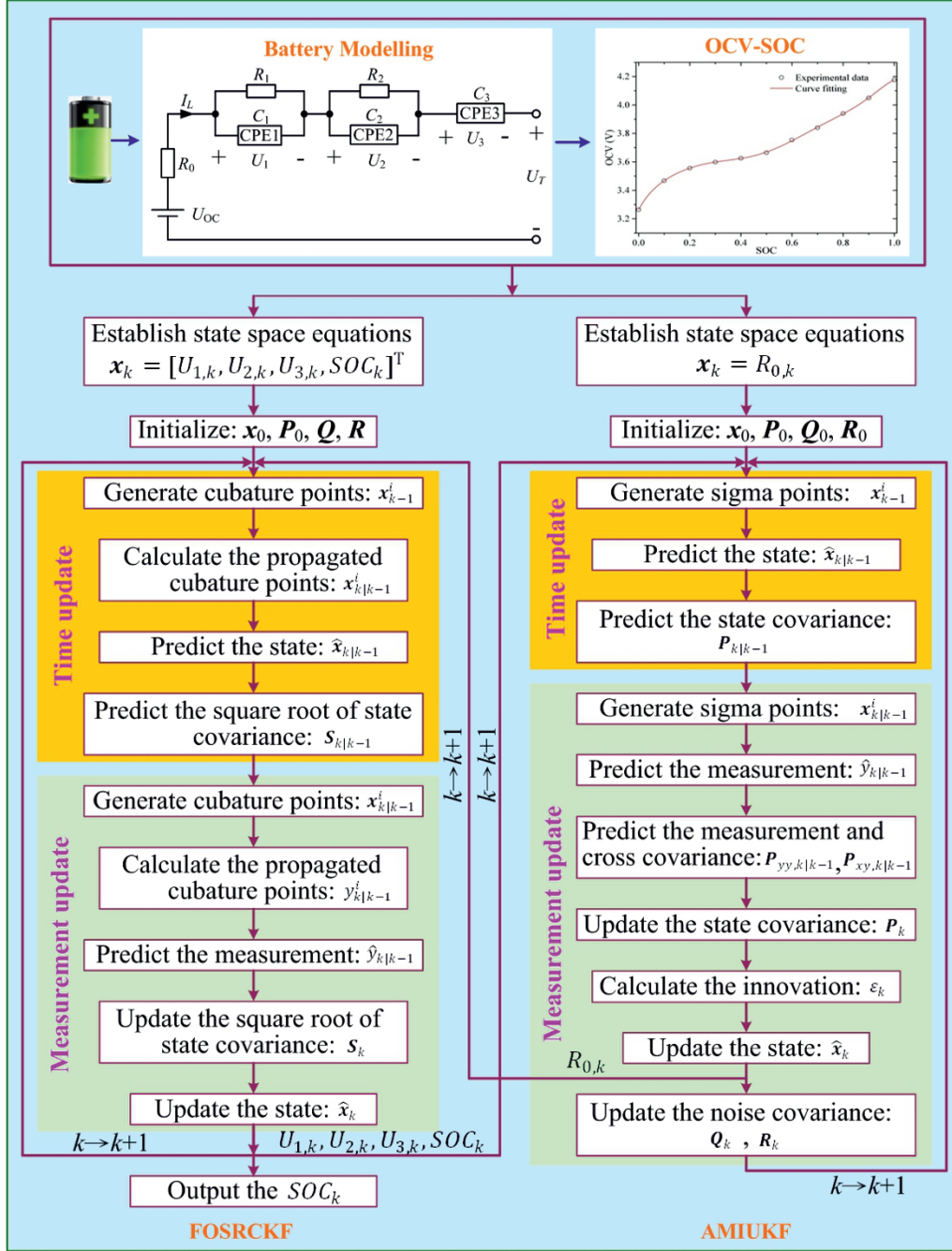


Fig. 1. Schematic of the proposed method

$$(2) \quad \binom{\alpha}{j} = \begin{cases} 1, & j = 0 \\ \frac{\alpha(\alpha-1) \cdots (\alpha-(j-1))}{j!}, & j > 0, \end{cases}$$

where $\mathfrak{D}$ is the differential operator, α is the differential order value, T_s is the sample time, and $[t/h]$ represents the integer part of t/h .

State space equations. According to Fig. 1, we can get the first four items of the following equations. Correspondingly, the SOC can be calculated as the last item of equation (3) according to the coulomb counting method.

$$(3) \quad \begin{cases} U_T(t) = U_{OC}(t) - R_0 I_L(t) - U_1(t) - U_2(t) - U_3(t) \\ \mathfrak{D}^{\alpha_1} U_1(t) = -\frac{1}{R_1 C_1} U_1(t) + \frac{1}{C_1} I_L(t) \\ \mathfrak{D}^{\alpha_2} U_2(t) = -\frac{1}{R_2 C_2} U_2(t) + \frac{1}{C_2} I_L(t) \\ \mathfrak{D}^{\alpha_3} U_3(t) = \frac{1}{C_3} I_L(t) \\ SOC(t) = SOC(0) - \frac{1}{C_p} \int_0^t I_L(t) dt, \end{cases}$$

where α_1 , α_2 and α_3 are the differentiation orders of the three CPEs, $0 < \alpha_1, \alpha_2, \alpha_3 < 1$, and C_p is the nominal battery capacity. In this study, the sample time, T_s , of the load current (or terminal voltage) is 1 s. According to equation (1), equation (3) can be transformed and discretized as

$$(4) \quad \begin{cases} U_{T,k} = U_{OC,k} - R_0 I_{L,k} - U_{1,k} - U_{2,k} - U_{3,k} \\ U_{1,k} = -\frac{1}{R_1 C_1} U_{1,k-1} + \frac{1}{C_1} I_{L,k-1} - \sum_{j=1}^L (-1)^j \binom{\alpha_1}{j} U_{1,k-j} \\ U_{2,k} = -\frac{1}{R_2 C_2} U_{2,k-1} + \frac{1}{C_2} I_{L,k-1} - \sum_{j=1}^L (-1)^j \binom{\alpha_2}{j} U_{2,k-j} \\ U_{3,k} = \frac{1}{C_3} I_{L,k-1} - \sum_{j=1}^L (-1)^j \binom{\alpha_3}{j} U_{3,k-j} \\ SOC_k = SOC_{k-1} - \frac{1}{C_p} I_{L,k-1}. \end{cases}$$

By defining $\mathbf{x}_k = [U_{1,k}, U_{2,k}, U_{3,k}, SOC_k]^T$, $u_k = I_{L,k}$, $y_k = U_{T,k}$, one can obtain the state space equations as follows:

$$(5) \quad \begin{cases} \mathbf{x}_k = \mathbf{A} \mathbf{x}_{k-1} + \mathbf{B} u_{k-1} - \sum_{j=1}^L (-1)^j \gamma_j \mathbf{x}_{k-j} + \mathbf{w}_k = f(\mathbf{x}_{k-1}, u_{k-1}) + \mathbf{w}_k \\ y_k = U_{OC}(x_{4,k}) - x_{1,k} - x_{2,k} - x_{3,k} - R_0 u_k + v_k = g(\mathbf{x}_k, u_k) + v_k, \end{cases}$$

where $x_{i,k}$ denotes the i th element of $\mathbf{x}_k$, $\mathbf{w}_k$ and v_k are independent white noise

with variances of $\mathbf{Q}_k$ and R_k ,

$$\begin{aligned}\mathbf{A} &= \text{diag} \left[-\frac{1}{R_1 C_1} \quad -\frac{1}{R_2 C_2} \quad 0 \quad 1 \right], \\ \mathbf{B} &= [1/C_1 \quad 1/C_2 \quad 1/C_3 \quad -1/C_p]^T, \\ \gamma_j &= \text{diag} \left[\begin{pmatrix} \alpha_1 \\ j \end{pmatrix} \quad \begin{pmatrix} \alpha_2 \\ j \end{pmatrix} \quad \begin{pmatrix} \alpha_3 \\ j \end{pmatrix} \quad 0 \right].\end{aligned}$$

Model parameters identification. An NCR18650 lithium-ion battery was operated under charging or discharging conditions to validate the proposed method. The battery has a rated capacity of 3.4 Ah and a rated voltage of 3.7 V. The current and voltage profiles under the Hybrid Pulse Power Characteristic (HPPC) test at 25 °C are utilized to identify model parameters as well as determine the relationship between the OCV and SOC. In this study, we adopt adaptive genetic algorithm (AGA) to identify model parameters, which detailed implementation of AGA can be found in our previous work [9]. The resulting parameter values of R_0 , R_1 , R_2 , C_1 , C_2 , C_3 , α_1 , α_2 and α_3 are 71 mΩ, 323 mΩ, 357 mΩ, 1104 F, 12 596 F, 75 023 F, 0.84, 0.92, 0.73, respectively.

Method. We design two Kalman filters, FOSRCKF and AMIUKF, for estimating the battery SOC and updating R_0 online. There is exchange of information recursively between FOSRCKF and AMIUKF at each step. The state values estimated by FOSRCKF, including SOC, U_1 , U_2 , and U_3 , are passed to AMIUKF. AMIUKF estimates the internal resistance R_0 of the battery and feeds back the estimated R_0 to FOSRCKF. Figure 1 shows the schematic of the proposed method.

Implementation of FOSRCKF. The state and measurement equations of FOSRCKF are shown in equation (5). The state vector $\mathbf{x}$ has four states to be estimated, namely SOC, U_1 , U_2 , and U_3 . FOSRCKF mainly includes two processes: time update and measurement update. The detailed implementation process of FOSRCKF is shown in Table 1, where $m = 2n$, n ($= 4$) is the dimension of $\mathbf{x}$, $\mathbf{I}$ is an identity matrix, ξ_i denotes the i -th column of matrix ξ , $i = 1, 2, \dots, m$, L is the memory length, which was ultimately determined to be 60 through trial and error, $\text{Tria}(\cdot)$ denotes the QR decomposition of the transpose of a matrix, $\mathbf{S}_k$ is the square root matrix of state covariance, $\mathbf{S}_{yy,k|k-1}$ is the square root matrix of measurement covariance, $\mathbf{P}_{xy,k|k-1}$ is the cross covariance matrix, and $\mathbf{K}_k$ is the Kalman gain.

Implementation of AMIUKF. AMIUKF is used for updating the internal resistance, R_0 , in real time. The state and measurement equations of AMIUKF are shown in the following equation

$$(6) \quad \begin{cases} \mathbf{x}_k = R_{0,k} = R_{0,k-1} + \mathbf{w}_k = f^*(\mathbf{x}_{k-1}, u_{k-1}) + \mathbf{w}_k \\ y_k = U_{OC}(SOC_k) - U_{1,k} - U_{2,k} - U_{3,k} - R_{0,k}u_k + v_k = g^*(\mathbf{x}_k, u_k) + v_k, \end{cases}$$

T a b l e 1
Process of FOSRCKF

Initialize $\mathbf{x}_0, \mathbf{P}_0, \mathbf{Q}, \mathbf{R}$, then <i>time update</i> :	
1. Generate the cubature points	$\mathbf{x}_{k-1}^i = \hat{\mathbf{x}}_{k-1} + \mathbf{S}_{k-1}\boldsymbol{\xi}_i, \boldsymbol{\xi} = \sqrt{m/2}[\mathbf{I}, -\mathbf{I}]$
2. Calculate the propagated cubature points	$\mathbf{x}_{k k-1}^i = f(\mathbf{x}_{k-1}^i, u_{k-1})$
3. Predict the state	$\hat{\mathbf{x}}_{k k-1}^* = \frac{1}{m} \sum_{i=1}^m \mathbf{x}_{k k-1}^i,$ $\hat{\mathbf{x}}_{k k-1}^* = \hat{\mathbf{x}}_{k k-1}^* - \sum_{j=1}^L (-1)^j \boldsymbol{\gamma}_j \hat{\mathbf{x}}_{k-j}$
4. Predict the square root of state covariance <i>measurement update</i> :	$\mathbf{x}_{k k-1}^* = \frac{1}{\sqrt{m}} [\mathbf{x}_{k k-1}^1 - \hat{\mathbf{x}}_{k k-1}^*, \dots, \mathbf{x}_{k k-1}^m - \hat{\mathbf{x}}_{k k-1}^*]$ $\mathbf{S}_{k k-1} = \text{Tria}([\mathbf{x}_{k k-1}^*, \mathbf{S}_Q]), \mathbf{Q} = \mathbf{S}_Q \mathbf{S}_Q^T$
5. Generate the cubature points	$\mathbf{x}_{k k-1}^i = \hat{\mathbf{x}}_{k k-1} + \mathbf{S}_{k k-1}\boldsymbol{\xi}_i$
6. Calculate the propagated cubature points	$y_{k k-1}^i = g(\mathbf{x}_{k k-1}^i, u_{k-1})$
7. Predict the measurement	$\hat{y}_{k k-1} = \frac{1}{m} \sum_{i=1}^m y_{k k-1}^i$
8. Predict the square root of measurement covariance	$\mathbf{Y}_{k k-1} = \frac{1}{\sqrt{m}} [y_{k k-1}^1 - \hat{y}_{k k-1}, \dots, y_{k k-1}^m - \hat{y}_{k k-1}]$ $\mathbf{S}_{yy,k k-1} = \text{Tria}([\mathbf{Y}_{k k-1}, \mathbf{S}_R]), \mathbf{R} = \mathbf{S}_R \mathbf{S}_R^T$
9. Predict the cross covariance	$\mathbf{X}_{k k-1} = \frac{1}{\sqrt{m}} [\mathbf{x}_{k k-1}^1 - \hat{\mathbf{x}}_{k k-1}, \dots, \mathbf{x}_{k k-1}^m - \hat{\mathbf{x}}_{k k-1}],$ $\mathbf{P}_{xy,k k-1} = \mathbf{X}_{k k-1} (\mathbf{Y}_{k k-1})^T$
10. Calculate the Kalman gain	$\mathbf{K}_k = \mathbf{P}_{xy,k k-1} [\mathbf{S}_{yy,k k-1} (\mathbf{S}_{yy,k k-1})^T]^{-1}$
11. Update the square root of state covariance	$\mathbf{S}_k = \text{Tria}([\mathbf{X}_{k k-1} - \mathbf{K}_k \mathbf{Y}_{k k-1}, \mathbf{K}_k \mathbf{S}_R])$
12. Update the state	$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k k-1} + \mathbf{K}_k (y_k - \hat{y}_{k k-1})$

where the parameter values of $SOC_k, U_{1,k}, U_{2,k}$ and $U_{3,k}$ come from FOSRCKF. Considering that internal resistance varies slowly as the battery cell ages; therefore, the state $R(0, k)$ is regarded as constant and disrupted by the state noise only.

AMIUKF also mainly includes two processes: time update and measurement update. The detailed implementation process of AMIUKF is shown in Table 2, where $\lambda = \alpha^2(n + \kappa) - n$, n ($= 1$) is the dimension of the state vector $\mathbf{x}$, α (set 0.01 in this study) is the scale factor, κ is zero, w_m^i and w_c^i are the weight. The last step of AMIUKF is updating the state and measurement noise covariance using Sage–Husa algorithm. The basic idea of Sage–Husa is using the innovation ε_k (i.e. the difference between the real measurement and predicted measurement) to estimate the state and measurement noise covariance, $\mathbf{Q}_k$ and R_k . In step 12, $\mathbf{C}_k = \frac{1}{l} \sum_{j=k-l-1}^k \varepsilon_j \left(\frac{1}{l} \sum_{j=k-l-1}^k \varepsilon_j \right)^T$, l is the length of the innovation, $d_k = (1 - b)/(1 - b^k)$, and b is the adaptive factor.

Considering that internal resistance varies slowly as the battery cell ages; therefore, the state $R_{0,k}$ is regarded as constant and disrupted by the state noise only.

Results and discussion. In order to verify the effectiveness and robustness of the proposed method (referred to as FOSRCKF-AMIUKF in this study), the

T a b l e 2

Process of AMIUKF

Initialize $\mathbf{x}_0, \mathbf{P}_0, \mathbf{Q}_0, \mathbf{R}_0$, then <i>time update</i> :	
1. Generate the sigma points	$\mathbf{x}_{k-1}^i = \hat{\mathbf{x}}_{k-1}, i = 0$ $\mathbf{x}_{k-1}^i = \hat{\mathbf{x}}_{k-1} + \sqrt{(n+\lambda)\mathbf{P}_{k-1}}, i = 1, 2, \dots, n$ $\mathbf{x}_{k-1}^i = \hat{\mathbf{x}}_{k-1} - \sqrt{(n+\lambda)\mathbf{P}_{k-1}}, i = n+1, \dots, 2n$
2. Predict the state	$\mathbf{x}_{k k-1}^i = f^*(\mathbf{x}_{k-1}^i, u_{k-1})$ $\hat{\mathbf{x}}_{k k-1} = \sum_{i=0}^{2n} w_m^i \mathbf{x}_{k k-1}^i$
3. Predict the state covariance	$\mathbf{P}_{k k-1} = \sum_{i=0}^{2n} w_c^i (\mathbf{x}_{k k-1}^i - \hat{\mathbf{x}}_{k k-1}) (\mathbf{x}_{k k-1}^i - \hat{\mathbf{x}}_{k k-1})^T$
<i>measurement update</i> :	
4. Generate the sigma points	$\mathbf{x}_{k k-1}^i = \hat{\mathbf{x}}_{k k-1}, i = 0$ $\mathbf{x}_{k k-1}^i = \hat{\mathbf{x}}_{k k-1} + \sqrt{(n+\lambda)\mathbf{P}_{k k-1}}, i = 1, 2, \dots, n$ $\mathbf{x}_{k k-1}^i = \hat{\mathbf{x}}_{k k-1} - \sqrt{(n+\lambda)\mathbf{P}_{k k-1}}, i = n+1, \dots, 2n$
5. Predict the measurement	$y_{k k-1}^i = g^*(\mathbf{x}_{k k-1}^i, u_{k-1}),$ $\hat{y}_{k k-1} = \sum_{i=0}^{2n} w_m^i y_{k k-1}^i$
6. Predict the measurement covariance	$\mathbf{P}_{yy,k k-1} = \sum_{i=0}^{2n} w_c^i (y_{k k-1}^i - \hat{y}_{k k-1}) (y_{k k-1}^i - \hat{y}_{k k-1})^T$
7. Predict the cross covariance	$\mathbf{P}_{xy,k k-1} = \sum_{i=0}^{2n} w_c^i (\mathbf{x}_{k k-1}^i - \hat{\mathbf{x}}_{k k-1}) (y_{k k-1}^i - \hat{y}_{k k-1})^T$
8. Calculate the Kalman gain	$\mathbf{K}_k = \mathbf{P}_{xy,k k-1} (\mathbf{P}_{yy,k k-1})^{-1}$
9. Update the state covariance	$\mathbf{P}_k = \mathbf{P}_{k k-1} - \mathbf{K}_k \mathbf{P}_{yy,k k-1} (\mathbf{K}_k)^T$
10. Calculate the innovation	$\varepsilon_k = y_k - \hat{y}_{k k-1}$
11. Update the state	$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k k-1} + \sum_{i=1}^p \lambda_i \mathbf{K}_{k-i+1} \varepsilon_{k-i+1}$
12. Update the noise covariance using Sage-Husa algorithm	$\mathbf{Q}_k = (1 - d_k) \mathbf{Q}_{k-1} + d_k \mathbf{K}_k \mathbf{C}_k (\mathbf{K}_k)^T$ $\mathbf{R}_k = (1 - d_k) \mathbf{R}_{k-1} + d_k [\mathbf{C}_k + \sum_{i=0}^{2n} w_c^i (y_{k k-1}^i - \hat{y}_{k k-1}) (y_{k k-1}^i - \hat{y}_{k k-1})^T]$

SOC estimation is conducted under the Urban Dynamometer Driving Schedule (UDDS) and the US06 Highway Driving Schedule. Another method for the SOC estimation, namely the square root cubature Kalman filter (SRCKF) is compared with FOSRCKF-AMIUKF. The SRCKF method is based on IOM instead of FOM, and it only estimates the battery SOC and does not update model parameters in real time. The SOC calculated by ampere hour integration method is regarded as the real SOC, which is compared with the estimated SOC.

Figure 2(a) shows the SOC estimation results of FOSRCKF-AMIUKF and SRCKF under the UDDS test. It can be observed that the SOC estimated by FOSRCKF-AMIUKF is closer to the real SOC compared with that estimated by SRCKF. The comparisons of SOC estimation error are shown in Fig. 2(b). The SOC estimation mean absolute error (MAE) and the root-mean square error (RMSE) are 0.96% and 1.17% for FOSRCKF-AMIUKF, whereas the same are 1.54% and 1.78% for SRCKF. Figure 2(c) and (d) show the comparisons of the terminal voltage estimation results and their errors. The terminal voltage estimation MAE and RMSE are 3.5 mV and 5.3 mV for FOSRCKF-AMIUKF, whereas the same are 9.1 mV and 12.5 mV for SRCKF. Obviously, FOMs can better describe

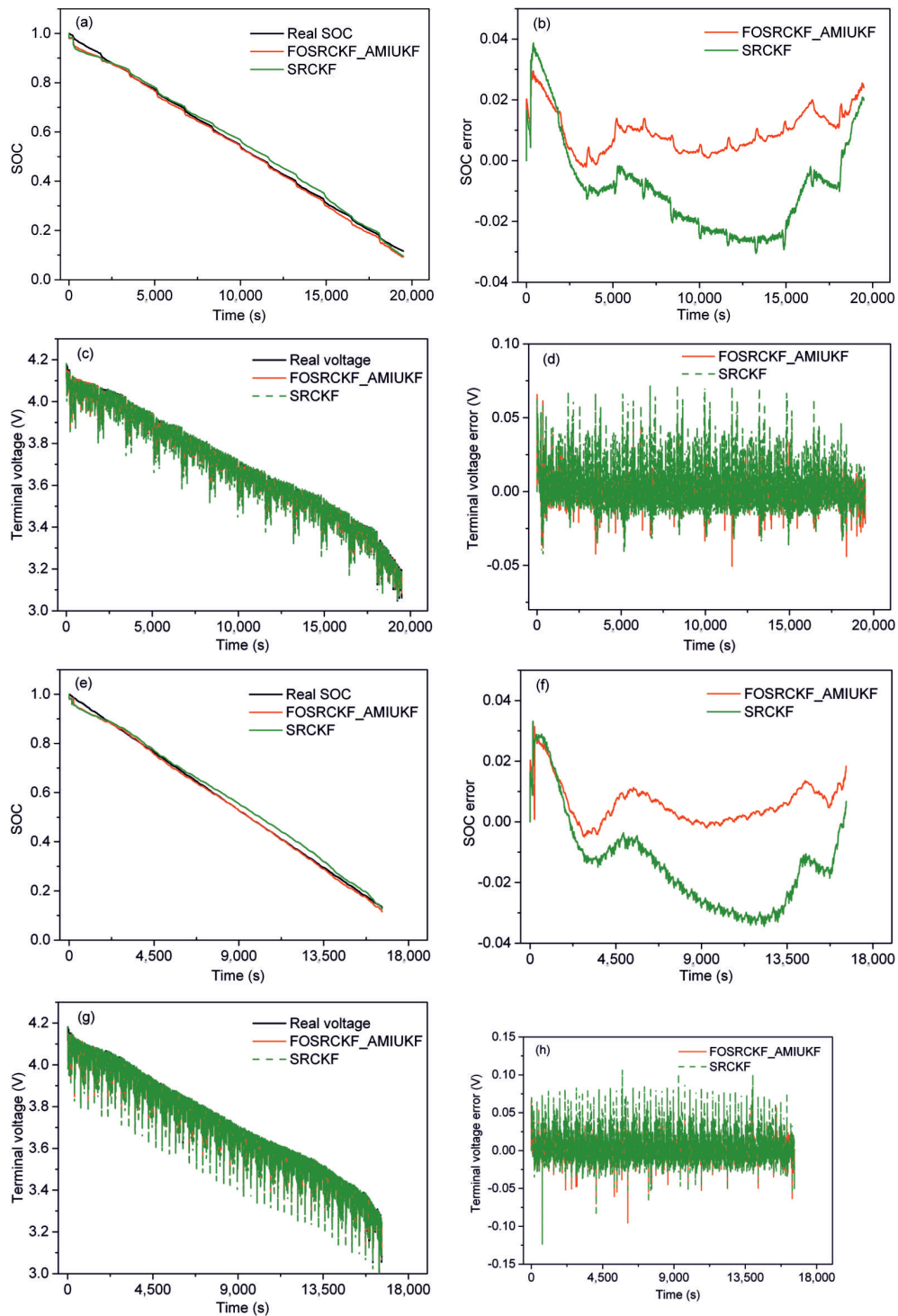


Fig. 2. Comparisons of the estimation results under the UDDS test and US06 test.
The UDDS test: (a)–(d); The US06 test: (e)–(h)

T a b l e 3

Estimation results of SOC and terminal voltage

Method	Test profile	SOC error (%)		Voltage error (mV)	
		MAE	RMSE	MAE	RMSE
FOSRCKF-AMIUKF	US06	0.66	0.89	4.20	6.60
	UDDS	0.96	1.17	3.50	5.30
SRCKF	US06	1.83	2.06	13.4	17.7
	UDDS	1.54	1.78	9.10	12.5

the internal behaviour of the battery compared with IOMs. Figure 2(e)–(h) show the SOC and terminal voltage estimation results under the US06 test. The MAE and RMSE of the SOC and terminal voltage estimation are shown in Table 3. From Figure 2(e)–(h) and Table 3, similar conclusions can be drawn to those under the UDDS test. These results demonstrate the effectiveness, accuracy and robustness of the proposed method.

Conclusion. The accuracy of battery modelling is crucial to model-based methods for the SOC estimation. In this study, a fractional order equivalent circuit model is adopted to replace the traditional integer order circuit model for battery modelling, which improves the model’s ability to describe the internal behaviour of the battery. A FOSRCKF-AMIUKF algorithm based on the fractional order model is proposed for the SOC estimation as well as online update of the battery internal resistance. Experimental results under the UDDS and US06 tests verify the effectiveness, accuracy and robustness of the proposed method. Compared with the SRCKF based on the integer order model, the FOSRCKF-AMIUKF has lower SOC estimation error and lower terminal voltage prediction error.

REFERENCES

- [1] WANG Y. J., J. Q. TIAN, Z. D. SUN et al. (2020) A comprehensive review of battery modeling and state estimation approaches for advanced battery management systems, *Renew. Sustain. Energy Rev.*, **131**, 110015.
- [2] XIONG B. Y., J. Y. ZHAO, Z. B. WEI, M. SKYLLAS-KAZACOS (2014) Extended Kalman filter method for state of charge estimation of vanadium redox flow battery using thermal-dependent electrical model, *J. Power Sources*, **262**, 50–61.
- [3] CHENG X., E. ZHANG, K. JIANG, K. L. WANG (2022) Dual fuzzy-based adaptive extended Kalman filter for state of charge estimation of liquid metal battery, *Appl. Energy*, **327**, 120091.
- [4] HOU J., J. W. LIU, F. W. CHEN et al. (2023) Robust lithium-ion state-of-charge and battery parameters joint estimation based on an enhanced adaptive unscented Kalman filter, *Energy*, **271**, 126998.
- [5] REZAEI O., A. RAHDAN, S. SARDARI et al. (2023) A fuzzy robust two-stage unscented Kalman filter method for uncertainty and state of charge estimation of lithium-ion batteries, *J. Power Sources*, **68**, 107883.

- [6] ZHU Q., M. G. XU, W. Q. LIU, M. Q. ZHENG (2019) A state of charge estimation method for lithium-ion batteries based on fractional order adaptive extended Kalman filter, *Energy*, **187**, 115800.
- [7] XIONG R., J. P. TIAN, W. X. SHEN, F. C. SUN (2019) A novel fractional order model for state of charge estimation in lithium ion batteries, *IEEE Trans. Veh. Technol.*, **68**, 4130–4139.
- [8] PETRAS I. (2011) Fractional Calculus. In: *Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation*, Heidelberg, Germany, Springer, 9–12.
- [9] LING L. Y., Y. WEI (2021) State-of-charge and state-of-health estimation for lithium-ion batteries based on dual fractional-order extended Kalman filter and on-line parameter identification, *IEEE Access*, **9**, 47588–47602.

Faculty of Engineering, Anhui Sanlian University, Hefei 230601, China
e-mail: weiyiing8ths@sohu.com