

MATHEMATICS

Group theory

FINITE GROUPS WHOSE ALL PROPER SUBGROUPS
HAVE ONLY IRREDUCIBLE CHARACTERS
OF SQUARE-FREE DEGREES

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Abstract

Lewis determined the structure of finite groups whose irreducible character odd-degrees are square-free. In this paper, we give an identification of the structure of non-solvable groups whose proper subgroups have only square-free degrees.

Key words: simple group, nonlinear irreducible character, proper subgroup

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1. Introduction. All groups under consideration are finite. Let $\text{Irr}(G)$ be the set of all complex irreducible characters of a finite group G and denote the set of character degrees of G by $\text{cd}(G)$, that is $\text{cd}(G) = \{\chi(1) : \chi \in \text{Irr}(G)\}$. Let $\text{SL}_2(q)$ ($\text{PSL}_2(q)$) denote the (projective) special linear group of degree 2 over a field of order q . And a *degree* always means an *irreducible character degree* in this paper.

In [1], HUPPERT and MANZ determined finite groups whose degrees are square-free. In particular, they showed that if G is non-solvable group with this property, then G has a section isomorphic to A_7 , where A_n is an alternating group

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of degree n . Also DOLFI, PACIFICI and SANUS [2] furthered Huppert and Manz's results and explored that if each of the real-valued irreducible character degrees is square-free, then G is either solvable or has a section isomorphic to A_7 ; see [2, Theorem 3.1]. Furthermore LEWIS and WHITE in [3] investigated the structure of non-solvable groups all of whose degrees are odd-square-free. Some scholars considered the relationship between the degrees of subgroups and the structure of finite group, for instance, HUMPHRIES and SKABELUND [4] determined the structure of metacyclic groups and LIU [5, 6] gave the structure of almost simple groups whose proper subgroups have either prime-power degrees or at most two real-valued degrees.

Let $\sum_p G$ and $\sum G$ be the set of proper subgroups and subgroups of a group G , respectively. We see that $G \in \sum G$, and $\sum G = \{G\} \cup \sum_p G$. Inspired by the above works [1–3], we introduce the following concept.

Definition 1.1. A group G is called a PD-group if for each $H \in \sum G$ and any $\chi \in \text{Irr}(H)$, $\chi(1)$ is square-free and a non-PD-group otherwise. A group G is named an SPD-group if for each $H \in \sum_p G$, H is a PD-group and a non-SPD-group otherwise.

We always think of an abelian group as a PD-group or an SPD-group.

Example 1.2. A group whose degrees are square-free does not mean that it is a PD-group or an SPD-group. For instance, let $G = A_7$, the alternating group of degree 7. It is easy to see from [7, p. 10] that

$$\text{cd}(A_7) = \{1, 2 \cdot 3, 2 \cdot 5, 2 \cdot 7, 3 \cdot 7, 3 \cdot 5, 5 \cdot 7\}.$$

Obviously all degrees of A_7 are square-free. Since $\text{PSL}_2(7)$ is a subgroup of A_7 and $8 \in \text{cd}(\text{PSL}_2(7))$, we have that $\text{PSL}_2(7)$ is a non-PD-group, so A_7 is neither a PD-group nor an SPD-group.

In comparison to Huppert and Manz's result [1], we first prove that a PD-group is solvable, namely the following result is obtained.

Theorem 1.3. *Let G be a finite PD-group. Then G is solvable.*

In Theorem 1.3, we consider all subgroups of which only have square-free degrees. A finite group whose degrees are square-free was first studied by Huppert and Manz in [1] and groups with this property are possibly non-solvable; see [1, Theorem 2.8]. Thus if we consider only the proper subgroups whose degrees are square-free, then we have this result.

Theorem 1.4. *Let G be a non-solvable SPD-group. Then G is isomorphic to one of the following groups:*

- (1) $\text{PSL}_2(q)$ for certain $q \in \{2^p, 3^p, r\}$ such that p, r are primes and $\frac{q-1}{\gcd(2, q-1)}$ is square-free.
- (2) $\text{SL}_2(3^p)$ with $3^p - 1$ is square-free.
- (3) $\text{SL}_2(q)$ with q an odd prime such that $q-1$ is square-free, and $q \equiv \pm 3, \pm 13 \pmod{40}$.

2. Minimal simple groups and basic properties of SPD-groups.

Lemma 2.1 ([8, Corollary 1]). *A simple group of composite order all of whose proper subgroups are solvable is isomorphic to one of the following simple groups:*

- (1) $\text{PSL}_2(2^p)$ for p a prime;
- (2) $\text{PSL}_2(3^p)$ for p an odd prime;
- (3) $\text{PSL}_2(p)$, for p any prime exceeding 3 such that $p^2 + 1 \equiv 0 \pmod{5}$;
- (4) $\text{Sz}(2^p)$ for p an odd prime;
- (5) $\text{PSL}_3(3)$.

Lemma 2.2. (1) *A group G is an SPD-group if and only if for each proper subgroup H of G , H is a PD-group.*

(2) *Let G be an SPD-group. Then for any non-trivial normal subgroup N of G , N and G/N are PD-groups.*

(3) *Let G be an SPD-group. If N is a normal subgroup of G , then G/N is an SPD-group.*

Proof. (1) We can get by the definition of an SPD-group.

(2) It is easy to see from $\text{Irr}(G/N) \subset \text{Irr}(G)$ and (1).

(3) It follows from [9, Corollary 11.29]. □

3. PD-groups and non-solvable SPD-groups. Let E_q be an elementary abelian $\pi(q)$ -group of order q , and C_n be a cyclic group of order n . Let $E_q : C_{q-1}$ or $q : (q-1)$ be a Frobenius group of order $q(q-1)$ with complement C_{q-1} or $(q-1)$ and kernel E_q or q , respectively. In a group G , denote by $\max G$ the set of maximal proper subgroups with regard to subgroup-order divisibility.

Proof of Theorem 1.3. Let G be a counterexample of minimal possible order. Then G is a nonsolvable group but its subgroups are solvable. So by Lemma 2.1, G is isomorphic to $\text{PSL}_2(q)$ for certain q , $\text{Sz}(2^p)$ or $\text{PSL}_3(3)$.

Consider $\text{PSL}_2(q)$.

Note that $\text{cd}(\text{PSL}_2(q))$ contains q as a member and that q is a prime power, so hypothesis shows that q is a prime. If $q = 2$, then $\text{PSL}_2(2) \cong S_3$ is solvable. If $q = 3$, then $\text{PSL}_2(3) \cong A_4$ is solvable too.

Observe that for $q \geq 4$, $q-1, q+1 \in \text{cd}(\text{PSL}_2(q))$ by [10]. So if $q \geq 5$ is a prime, then one of the numbers $q-1$ and $q+1$ is divisible by 4, a contradiction.

Consider $\text{Sz}(2^p)$. Then $2^{2p} \in \text{cd}(\text{Sz}(2^p))$ by [11, Theorem 5], a contradiction.

Consider $\text{PSL}_3(3)$. Then by [7, p. 13], $2^4 = 16 \in \text{cd}(\text{PSL}_3(3))$, a contradiction.

It follows that the minimal counterexample does not exist, so G is solvable. □

Corollary 3.1. *Let G be a non-solvable SPD-group. Then G is a non-PD-group.*

Proof. Since G is non-solvable, there is a section K/H isomorphic to a non-abelian simple group. Note that K/H is a normal subgroup of G/H , and that G/H is an SPD-group, so K/H is a PD-group. By Theorem 1.3, K/H is solvable, a contradiction. $\square$

Theorem 3.2. *Let G be a non-abelian simple SPD-group. Then G is isomorphic to $\text{PSL}_2(q)$, where q is satisfying one of the following conditions:*

- (1) $q = 2^f$, and $q - 1$ is square-free with f a prime;
- (2) $q = 3^p$ and $\frac{q-1}{2}$ is square-free with p an odd prime.
- (3) $q = p$ is a prime, $\frac{q-1}{2}$ is square-free with $q = p \not\equiv \pm 1 \pmod{10}$.

Proof. A PD-group is solvable by Lemma 1.3. From the definition of an SPD-group, it follows that the proper subgroups of a group are solvable. Now Lemma 2.1 gives that G is possibly isomorphic to $\text{PSL}_2(2^p)$ for p a prime; $\text{PSL}_2(3^p)$ for p an odd prime; $\text{PSL}_2(p)$, for p any prime exceeding 3 such that $p^2 + 1 \equiv 0 \pmod{5}$; $Sz(2^p)$ for p an odd prime; $\text{PSL}_3(3)$. In what follows, we will analyze all these cases.

Case 1. $\text{PSL}_2(2^p)$ for p a prime.

In this case $\text{PSL}_2(2^p)$ contains $E_q : C_{q-1}$, $D_{2(q-1)}$, and $D_{2(q+1)}$ as its maximal subgroups. Observe that $\text{cd}(E_q : C_{q-1}) = \{1, q-1\}$, and $\text{cd}(D_{2(q\pm 1)}) = \{1, 2\}$, so hypothesis shows that $q - 1 = 2^p - 1$ is square-free. Thus $\text{PSL}_2(2^p)$ is an SPD-group with $2^p - 1$ square-free.

Case 2. $\text{PSL}_2(3^p)$ for p an odd prime.

Now Table 1 shows that $\max \text{PSL}_2(3^p) = \{E_q : C_{(q-1)/2}, D_{q+1}, D_{q-1}\}$. Note that $A_5 \notin \max \text{PSL}_2(3^p)$ (in fact, we see that $3^4 \equiv 1 \pmod{10}$). Since p is an odd prime, $p = 4r + 1$ or $p = 4r + 3$ say, we get that $q = 3^p \equiv \pm 3 \pmod{10}$, so $q \equiv \pm 1 \pmod{10}$ has no solution in $\mathbb{N}$, the set of positive integers). We know that $\frac{3^p-1}{2} \in \text{cd}(E_{3^p} : C_{(3^p-1)/2})$ and so $\frac{3^p-1}{2}$ is square-free by hypothesis. Therefore $\text{PSL}_2(3^p)$ is an SPD-group if $(3^p - 1)/2$ is square-free with p a prime.

T a b l e 1

$\text{PSL}_2(q)$, $q \geq 5$ [12, Chap. III, Theorem 8.27]

	$\max \text{PSL}_2(q)$	Condition
$\bar{C}_1$	$E_q : C_{(q-1)/m}$	$m = \gcd(q-1, 2)$
C_2	$D_{2(q-1)/m}$	$q \notin \{5, 7, 9, 11\}$
C_3	$D_{2(q+1)/m}$	$q \notin \{7, 9\}$
C_5	$\text{PSL}_2(q_0).(m, b)$	$q = q_0^b$, b a prime, $q_0 \neq 2$
C_6	S_4	$q = p \equiv \pm 1 \pmod{8}$
	A_4	$q = p \equiv 3, 5, 13, 27, 37 \pmod{40}$
S	A_5	$q \equiv \pm 1 \pmod{10}$, $F_q = F_p[\sqrt{5}]$

Case 3. $\text{PSL}_2(p)$ for p any prime exceeding 3 such that $p^2 + 1 \equiv 0 \pmod{5}$.

If $p = 5$, then $\text{PSL}_2(5) \cong \text{PSL}_2(4)$ is considered. If $p = 7$, then by [7, p. 3], $\max \text{PSL}_2(7) = \{S_4, 7 : 3\}$; if $p = 11$, then by [7, p. 7], $\max \text{PSL}_2(11) = \{A_5, 11 : 5, D_{12}\}$. If $p \geq 13$ is odd prime, then by Table 1, we get that $\max \text{PSL}_2(p)$ has possibly $E_p : C_{(p-1)/2}$, D_{p+1} , D_{p-1} , S_4 , A_4 and A_5 as its members. If $p \equiv \pm 1 \pmod{10}$, then $A_5 \in \max \text{PSL}_2(p)$ and A_5 is a non-PD-group. Thus $p \equiv \pm 3 \pmod{10}$ as p is a prime. Meanwhile, $E_p : C_{(p-1)/2} \in \max \text{PSL}_2(p)$, so by assumption, $\frac{p-1}{2}$ is square-free. Since $p^2 + 1 \equiv 0 \pmod{5}$, we have $p \equiv \pm 3 \pmod{5}$. It follows that $\text{PSL}_2(p)$ is an SPD-group when p is a prime such that $\frac{p-1}{2}$ is square-free and $p \equiv \pm 3 \pmod{10}$.

Case 4. $Sz(2^p)$ for p an odd prime.

In this case, by Table 2, $C_{2^p \pm \sqrt{2 \cdot 2^p + 1}} : C_4 \in \max Sz(2^p)$, so $2^2 = 4 \in \text{cd}(C_{2^p \pm \sqrt{2 \cdot 2^p + 1}} : C_4)$, a contradiction. So $Sz(2^p)$ is a non-SPD-group.

T a b l e 2

$Sz(q)$, $q = 2^{2m+1}$ ([13, p. 385])

$\max(Sz(q))$	Condition
$E_{q^2} : C_{q-1}$	$q = q_0^a$, a prime, $q_0 \geq 8$
$D_{2(q-1)}$	
$C_{q+\sqrt{2q}+1} : C_4$	
$C_{q-\sqrt{2q}+1} : C_4$	
$Sz(q_0)$	

Case 5. $\text{PSL}_3(3)$.

By [7, p. 13], $3^2.2.S_4 \in \max \text{PSL}_3(3)$ and by [14], $\text{cd}(3^2.2.S_4) = \{1, 2, 3, 4, 8, 16\}$. Thus $\text{PSL}_3(3)$ is a non-SPD-group. $\square$

Now we will give

Proof of Theorem 1.4. As G is non-solvable, [15, Lemma 2] shows that G has a normal sequel $1 \leq H \leq K \leq G$ such that K/H is isomorphic to the direct product of isomorphic non-abelian simple groups and $|G/K|$ divides $|\text{Out}(K/H)|$, where $\text{Out}(A)$ denotes the outer-automorphism group of a group A . We can get from Lemma 2.2(3) that K/H is an SPD-group, so K/H is isomorphic to

$$\underbrace{S \times S \times \cdots \times S}_{k \text{ times}}$$

where S is isomorphic to the groups listed as in Theorem 3.2. If $k \geq 2$, then by [9, Theorem 4.21], K/H is a non-SPD-group (in fact, we can choose a subgroup of the form

$$S \times S \times \cdots \times S \times C_3 \text{ from } S \times S \times \cdots \times S.$$

Observe that $S \times S \times \cdots \times S \times C_3$ is non-solvable, so by Corollary 3.1, $S \times S \times \cdots \times S \times C_3$ is a non-PD-group). Thus K/H is isomorphic to a non-abelian simple group. Now

we have $S \leq G/H \leq \text{Aut}(S)$, where $\text{Aut}(A)$ is the automorphism group of a group A . We see from Corollary 3.1 that for any M with $S < M \leq \text{Aut}(S)$, M has a non-PD-subgroup S . It follows that G/H is isomorphic to $\text{PSL}_2(q)$ for certain q . In what follows, three cases are investigated.

Case 1. q is a 2-power.

Then the fact that $\text{GL}_2(q) \cong \text{SL}_2(q) \cong \text{PSL}_2(q)$ gives that $H \cap C_G(\text{PSL}_2(q)) = 1$, so G is isomorphic to $H \times \text{PSL}_2(q)$. If $H \neq 1$, then G has $\text{PSL}_2(q)$ as its subgroup. By Corollary 3.1, $\text{PSL}_2(q)$ is a non-PD-group, a contradiction. So $G \cong \text{PSL}_2(q)$. By Table 1, we know that $\max \text{PSL}_2(q) = \{E_q : C_{q-1}, D_{2(q+1)}, D_{2(q-1)}\}$, so hypothesis shows that $q-1$ is square-free. Thus $\text{PSL}_2(q)$ is an SPD-group if $q = 2^p$, and $2^p - 1$ is a prime.

Case 2. $q = 3^r$ with r an odd prime.

Now $G'H/H$ is isomorphic to $\text{PSL}_2(3^r)$. We know from [7, p. xvi] that the order of the Schur multiplier of $\text{PSL}_2(3^r)$ is two, so $G' \cap H \leq C_2$. If $G' \cap H = 1$, then $G' \cong \frac{G'}{G' \cap H} \cong \frac{G'H}{H} \cong \text{PSL}_2(3^r)$, so $G \cong \text{PSL}_2(3^r) \times H$. Obviously $H = 1$ as above arguments. If $G' \cap H = C_2$, then G is isomorphic to $\text{SL}_2(3^r)H$. Also we see that $H = C_2$ (note that $C_2 = Z(\text{SL}_2(3^r))$), for otherwise, $\text{SL}_2(3^r)$ is a proper subgroup of G and so it is a PD-subgroup, but this is impossible as $\text{PSL}_2(3^r) \cong \frac{\text{SL}_2(3^r)}{Z(\text{SL}_2(3^r))}$ and $\text{Irr}(\text{PSL}_2(3^r)) \subseteq \text{Irr}(\text{SL}_2(3^r))$. Now G is isomorphic to $\text{SL}_2(3^r)C_2 = \text{SL}_2(3^r)$. Since r is an odd prime, we have that by Table 3, $\text{SL}_2(3^r)$ has $E_{3^r} : C_{3^r-1}$, $Q_{2(3^r-1)}$, $Q_{2(3^r+1)}$, $\text{SL}_2(3)$ and $\text{PSL}_2(3)$ as its subgroups. Observe that $\text{cd}(E_{3^r} : C_{3^r-1}) = \{1, 3^r - 1\}$. Thus $\text{SL}_2(3^r)$ is an SPD-group if $3^r - 1$ is square-free for an odd prime r .

T a b l e 3

$\text{SL}_2(q)$, $q \geq 4$ ([13, p. 377])

$\max(\text{SL}_2(q))$	Condition
$E_q : C_{q-1}$	
$Q_{2(q-1)}$	$q \neq 5, 7, 9, 11$; q odd N1 if $q = 7, 11$; N2 if $q = 9$
$D_{2(q-1)}$	q even
$Q_{2(q+1)}$	$q \neq 7, 9$; q odd N1 if $q = 7$; N2 if $q = 9$
$D_{2(q+1)}$	q even
$\text{SL}_2(q_0).2$	$q = q_0^2$, q odd
$\text{SL}_2(q_0)$	$q = q_0^r$, q odd, r odd prime
$\text{PSL}_2(q_0)$	$q = q_0^r$, q even, $q_0 \neq 2$, r prime
$2.S_4$	$q = p \equiv \pm 1 \pmod{8}$
$2.A_4$	$q = p \equiv \pm 3, 5, \pm 13 \pmod{40}$ N1 if $q = p \equiv \pm 11, \pm 19 \pmod{40}$
$2.A_5$	$q = p \equiv \pm 1 \pmod{10}$ $q = p^2$, $p \equiv \pm 3 \pmod{10}$

where N1 maximal under $\langle \delta \rangle$ with $|\delta| = (q-1, 2)$;

N2 maximal under subgroups not contained in $\langle \varphi \rangle$ with $|\varphi| = e$, $q = p^e$, p a prime.

Case 3. $q \geq 5$ is a prime.

Then $G'H/H \cong \text{PSL}_2(q)$. Note that the order of the Schur multiplier of $\text{PSL}_2(q)$ is 2, so $G' \cap H \leq C_2$. If $G' \cap H = 1$, then $G' \cong \text{PSL}_2(q)$, so $G \cong \text{PSL}_2(q) \times H$ as above arguments. It is easy to get $H = 1$. If $G' \cap H \cong C_2$, then G is isomorphic to $\text{SL}_2(q)H$. Also we have $H = C_2$ as in Case 2. We see from Table 3 that $\max \text{SL}_2(q)$ possibly contains $E_q : C_{q-1}$, $Q_{2(q \pm 1)}$, $2.S_4$, $2.A_4$ and $2.A_5$ as its members. We see that $q-1 \in \text{cd}(E_q : C_{q-1})$, so by hypothesis, $q-1$ is square-free. If $q \equiv \pm 1 \pmod{8}$, then $2.S_4 \in \max \text{PSL}_2(q)$; if $q \equiv \pm 1 \pmod{10}$, then $2.A_5 \in \max \text{PSL}_2(q)$. Observe that $2.A_5$ and $2.S_4$ are non-PD-groups because $\text{cd}(2.A_5) = \{1, 2, 3, 4, 5, 6\}$ and $\text{cd}(2.S_4) = \{1, 2, 3, 4\}$ by [14]. It follows that $q \equiv \pm 3, \pm 13 \pmod{40}$. Therefore $\text{SL}_2(q)$ is an SPD-group if q is a prime such that $q-1$ is square-free, and $q \equiv \pm 3, \pm 13 \pmod{40}$. $\square$

Theorem 3.3. *Let G be finite SPD-group which has no section isomorphic to $\text{PSL}_2(q)$. Then G is solvable.*

Proof. Assume the contrary, that G is a nonsolvable SPD-group, and so by Theorem 1.4, G is isomorphic to

- (1) $\text{PSL}_2(q)$ for certain $q \in \{2^p, 3^p, r\}$ such that p, r are primes and $\frac{q-1}{\gcd(2, q-1)}$ is square-free.
- (2) $\text{SL}_2(3^p)$ with $3^p - 1$ square-free;
- or
- (3) $\text{SL}_2(q)$ with q an odd prime such that $q-1$ is square-free, and $q \equiv \pm 3, \pm 13 \pmod{40}$.

It follows that G has a section isomorphic to $\text{PSL}_2(q)$, a contradiction. $\square$

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