

APPLYING A SITUATIONAL ANALYSIS SOLVER TO PROBLEMS INVOLVING UMBRA/SUNLIT MODEL

Atanas Marinov Atanassov

Received on May 17, 2023
Presented by P. Getsov, Corresponding Member of BAS, on May 30, 2023

Abstract

The search for optimal time intervals for satellite operations performance is based on verifying specific conditions of geometric or physical nature. Several conditions are combined in a situational problem. To be able to solve different situational problems, it is necessary to develop program codes for verifying different situational conditions in advance. Two situational conditions for determining if the satellite is in the sunlit zone or the umbra based on the conical models of the Earth's shadow are presented. Problems with one or more conditions are solved by a developed parallel solver for situational analysis. Conditions checking the position of a satellite relative to the Earth's shadow can be combined with other situational conditions.

Key words: constraints analysis, conic shadow model

Introduction. Current trends in the development of satellite technologies lead on the one hand to the use of smaller satellites and on the other hand, to multi-satellite missions instead of large multi-purpose satellites [1,2]. The process of space missions' analysis and design is related to the clarification and particularization of many and different parameters linked to scientific instruments, satellite subsystems, orbital parameters or templates of multi-satellite systems. Different kinds of optimization tasks are solved several times at large time intervals based on computer simulation. For that reason, the preparation of satellite missions is time-consuming.

DOI:10.7546/CRABS.2023.07.09

Computer simulations play an increasing role at all stages of preparation and operational implementation of multi-satellite missions. The application of parallel algorithms and calculations is crucial in solving large-scale problems involving hundreds and thousands of satellites.

An essential part of the space missions' analysis is the so-called situational analysis. The situational analysis deals with determining optimal time intervals, suitable for the implementation of satellite operations related to measurements and real-time experiments. The determination of such time intervals demands knowing the satellite trajectories in consecutive moments and checking different situational conditions. The situational conditions are related to the objects of investigation (disposed on a planetary surface, celestial sphere or specific place of space), visibility and illumination of the objects, environmental parameters (magnetic or electric fields, densities of different atmospheric ingredients, or radiation background). This type of analysis is applied to different stages of missions' preparation, starting with the conceptual study and preliminary analysis, going through missions' definition, design and development, and finishing with implementation.

The time intervals' determination of satellites' passage across the Earth's umbra as well as across the sunlit zone has wide practical applications. There are several main groups of tasks. One is related to assessments of the possibility of energy generation needed for the operation of satellites, using solar arrays. Another is related to the conditions for conducting observations of objects and regions on the Earth's surface. In some cases, they are made in the sunlit zone (in the visible spectral region) and in other cases, they are made when the satellite is in Earth's umbra (in the infrared spectral region, objects on the sky sphere). In order to carry out detailed analyses, suitable numerical algorithms and computational tools are required. WERTZ and LARSON [3] emphasize that mission analysis algorithms must be simple and effective enough to allow multiple runs. The development of effective methods and computer programs for situational analysis is very important when many problems need to be solved, each involving more than one situation.

Algorithms and calculation tools for space mission analysis and design are under development at the Space Research and Technology Institute at the Bulgarian Academy of Sciences. One such tool is a parallel solver for situational analysis [4]. Algorithms and program realization of the conical model of the Earth's shadow are discussed in the present work.

Related work. The use of geometric models of the Earth's shadow in the analysis, design or satellite operation scheduling of space missions is related to determining the motion of the satellites. The analytical solution of the equation of undisturbed satellite motion had wide applications in the early years of the space age. It is used in the development of analytical methods for determining the entry and exit from the Earth's shadow.

Analytical methods. The development of geometric models of the Earth's

shadow and the determination of the moments when satellites enter and exit from it started at the very beginning of the space age. The first known publications that present calculation algorithms for these moments are based on an analytical (Kepler's) solution of the equation of satellite motion [5–8].

The motion of the satellite in the plane of Kepler's orbit is considered undisturbed for at least a certain time interval (for example, it may be the time for one revolution around the central body). This type of motion in Kepler's plane is described by a transcendental equation known as the Kepler equation [9]. The intersection of the orbital plane (Kepler plane) with the geometric model of the Earth's shadow (cylindrical or conical surface) is a second-row plane curve – quadric. The two equations of the corresponding curve in an orbital coordinate system and the Kepler equation, represent a system that is the subject of study by various authors [6,8].

The analytical approach is very efficient because the calculation procedure is applied only once for the entire period in which satellite orbital parameters and umbra model parameter (radius vector of the Sun) are constants. It is necessary to correct the orbital parameters at the end of some period (one revolution). The radius vector of the Sun can be calculated for the same period of time. However, there are some drawbacks. First, the accuracy of this approach is limited by the accuracy of the Kepler motion. Furthermore, not many other situational conditions can be represented by geometric models, whose approach can be applied to determine the corresponding time intervals when those conditions are met. A situational condition related to the Earth's shadow cannot be combined with other conditions not represented by geometric models.

Analysis based on numerical integration of satellite orbits. Computer simulations of satellite missions' functionality are based on satellites' motion models and situational analysis. The motions of satellites are integrated through an appropriate numerical or analytical method. The situational problems are solved at each step by checking respective diverse conditions.

The energy production from the Sun and the design of the energy satellite subsystem are considered by YANG et al. [10] and IVANUSHKIN et al. [11]. The various satellite subsystems (service and payload) are designed to perform certain operations and have specific energy needs. First of all, the Sun is the primary source of energy that determines the initial conditions for the design of a satellite energy system based on solar arrays. The scientific tasks to be solved with the satellite mission determine the choice of orbital parameters. Orbital parameters and parameters of the Earth's shadow predetermine the operating conditions of the energy system. The energy generated in the lit zone is distributed not only for the basic subsystems but also for the performance of planned operations (service and subscribed operations). The unused energy goes for charging rechargeable batteries, and then rechargeable batteries are used as an energy source to meet all needs. Wertz and Larson [3] and KURENKOV [12] consider different aspects of the

enumerated problems above. Another important practical problem is related to the temperature changes of satellite solar arrays. LI et al. [13] developed a model and conducted a thermal analysis of the solar arrays of GEO and LEO satellites based on the conic umbra model.

Problem definition. Concept of situation analysis. As pointed out above, situational analysis deals with solving problems related to satellite operation optimization according to necessary restrictive conditions. Each situational problem SP is composed of one or a conjunction of several situational conditions sc_i of different types:

$$(1) \quad SP = sc_1 \wedge sc_2 \wedge \dots \wedge sc_n.$$

The conditions themselves are predicate functions accepting values of one or zero. They can be generally represented as:

$$sc_i = sc_i(\{\vec{R}\}, \{\alpha\}, \{\beta\}, t,$$

where $\{\vec{R}\}$ is a set of radius vectors of objects, $\{\alpha\}$ is a set of parameters of the mathematical model describing the situational condition, and $\{\beta\}$ is a set of constraints that are specific to the conditions. The calculation of the SP function is done by sequentially checking the conditions sc_i . The application of Horner's rule allows cancelling the verification of the conditions when an unfulfilled condition sc_i is found:

$$(2) \quad SP = (\dots(\dots(sc_1 \wedge sc_2) \wedge \dots) \wedge sc_l \dots \wedge sc_{n-1}) \wedge sc_n.$$

It is important to keep in mind that each condition sc_i is satisfied within the time interval $T_{i,k}$ and is not fulfilled for the next adjacent interval $T_{i,k}^*$. The only requirement to $T_{i,k}$ and $T_{i,k}^*$, which will be taken into account, are $T_{i,k}, T_{i,k}^* \gg \Delta t$, where t is a step in simulation time. The index k denotes the serial number of the interval within the time horizon.

Parallel solving of situational problems. Advances in computer technologies provide new opportunities for the compilation of even more complex simulation models but also place demands on their development. The article comments on the solving of situational problems in the context of parallel calculations (using multiprocessor and/or multicore systems). This requires special approaches and program models in the development of computational algorithms. The reasons for applying parallel calculations are the following:

1. complex mathematical models are used for the calculation of different situational conditions when searching for orbital events;
2. simultaneous solving of many situation problems is included in a simulation model;

3. the simulation time intervals ($t_{begin}..t_{end}$) are large;
4. repeatedly solving of situation model based on situational analysis for space mission parameters optimization (concerning satellites, instruments, ground-based stations' and objects of investigations).

All compiled situational problems can be applied to all members in a multi-satellite system. The number of situational problems needed to be solved may increase according to the number of satellites, objects for observation and scientific problems.

Geometric model of umbra and penumbra. A geometric model that is used for calculating the parameters of the Earth's shadow is shown in Fig. 1. The circles (O_s, R_s) and (O_E, R_E) represent the Sun and the Earth in a plane passing through the line $\overleftrightarrow{l}$ and point S where the satellite is located, respectively. The radii of the Earth R_E and the Sun R_s are assumed to be known. The radius vector $\vec{R}_{SE}(t)$ of the Sun in a geo-equatorial coordinate system is determined based on methods of astrodynamics. The radius vector of the satellite $\vec{r}_{sat}(t)$ is also determined based on numerical or analytical methods.

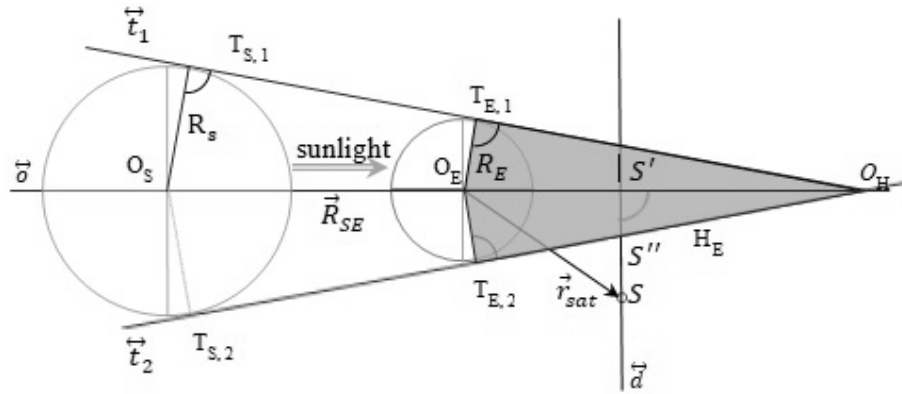


Fig. 1. Sun-Earth's umbra cone geometry

The lines $\overleftrightarrow{t}_1$ and $\overleftrightarrow{t}_2$ are tangent at the points ($T_{S,1}, T_{S,2}$) and ($T_{E,1}, T_{E,2}$) to the Sun and the Earth, respectively, in a plane defined by the vectors $\vec{R}_{SE}$ and $\vec{r}_{sat}$. From the similarity of the triangles $\Delta O_S T_{S,2} O_H$ and $\Delta O_E T_{E,2} O_H$ a formula can be derived for the height of the conical shadow of the Earth:

$$H_E = \frac{R_E \cdot |\vec{R}_{SE}|}{R_S - R_E}.$$

In this formula, R_E denotes the Earth's radius, the distance between the Sun and the Earth (the magnitude of the vector $\vec{R}_{SE}$) varies during the year in the interval

from 147 099 760 km to 152 104 285 km. The height of the Earth's shadow H_E can be assumed to be a slowly changing quantity.

The next step is to determine whether the satellite is in the shadow of the Earth. One condition for this is that the sub-satellite point is located in the unlit part of the Earth's surface. It is equivalent to checking if $\angle O_H O_E T_{E,2} < \angle O_H O_E S$:

$$\frac{\vec{R}_{SE} \cdot \vec{r}_{sat}}{|\vec{R}_{SE}| \cdot |\vec{r}_{sat}|} > \frac{\overline{O_E T_{E,2}}}{H_E},$$

because it compares the cosines of the indicated angles on both sides of the inequality without the use of arccosines.

To determine whether the satellite is in the shadow of the Earth, it remains to compare the segments $\overline{S'S''}$ and $\overline{S'S}$ on the line $\overleftrightarrow{d}$, perpendicular to the line $\overleftrightarrow{O'S}$. From the similarity of the triangles $\Delta O_H S' S''$ and $\Delta O_H T_{E,2} O_E$ it follows:

$$\overline{S'S''} = R_E \cdot \frac{H_E - \overline{O_E S'}}{\overline{T_{E,2} O_H}} = R_E \cdot \frac{H_E - |\vec{r}_{sat}| \cdot \cos(\angle O_H O_E S)}{\sqrt{H_E^2 - R_E^2}}.$$

The length of the line segment $\overline{T_{E,2} O_H}$, a leg in the right triangle $\Delta O_H T_{E,2} O_E$, is determined based on the length of the Earth's shadow H_E and the magnitude of the radius of the Earth R_E , or finally:

$$\overline{S'S''} = R_E \cdot \frac{H_E - \vec{e}_{SE} \vec{r}_{sat}}{\sqrt{H_E^2 - R_E^2}},$$

where $\vec{e}_{SE}$ is the unit radius vector of the Sun.

The length of $\overline{S'S}$ is determined by $\Delta O_E S' S$:

$$\overline{S'S} = |\vec{r}_{sat}| \cdot \sin(\angle S' O_E S)$$

or

$$\overline{S'S} = r_{sat} \cdot \sqrt{1 - \left(\frac{\vec{R}_{SE} \cdot \vec{r}_{sat}}{|\vec{R}_{SE}| \cdot |\vec{r}_{sat}|} \right)^2}.$$

It is important not to forget that determining the length of $\overline{S'S''}$ makes sense only if $\overline{O_E S'} < H_E$.

Program realization. A situational problem description model. A descriptor of situational problems is a one-dimensional array whose elements are derived types containing the values of different attributes (parameters and constraints as well as results) of the conditions comprised in the problem [14]. The first (zero) element of the descriptor contains control information and results about the entire situational problem. The descriptors of different situational problems

are combined into a two-dimensional array with dimensions $K \times N$, where N is the number of problems and K is the maximum number of situational conditions among all problems.

Design of the computer subroutines for the umbra/sunlit situational conditions. Two computer subroutines **Sit__78** and **Sit__79** have been developed for checking if the satellite is in the shadow, or the sunlight zone, respectively [14]. The subroutines are realized in Fortran language. When carrying out a situational analysis, which is related to the simulation of multi-satellite systems, some problems refer to observations in the Earth's shadow and others to observations in the sunlit zone.

In simulations of multi-satellite missions, some calculations about the geometry of the shadow are the same for all satellites. For this reason, the subroutine named **Preliminary_Calculations** is added to optimise computations. It performs some calculations related to the umbra and sunlit zones that depend on the Sun-Earth distance.

Parallel situational analysis solver. A simulation of multi-satellite missions with many objects of observation requires solving many situational problems. Repeated simulations to optimize different parameters also lead to the need for multiple situational analyses. The functionality and reliability of multi-satellite systems are a result of the space environment's impact on the subsystems and the scientific instruments over a long time horizon. The computer simulations clarifying changes in the functionality and the reliability require computational efficiency of situational analysis. The application of parallel calculations is a step in the right direction.

Different situational conditions based on specific computational models give rise to a different number of computational operations. Such models are irregular and an imbalance occurs when the calculations are parallelized. This is due to the uneven distribution of computational operations between the available processors. In case of poor distribution, some processors complete their problems and are free, while others continue their calculations. The "Pool of threads" model copes with this problem [15].

A parallel solver for situational analysis was developed for this purpose. It is a processing program that consistently checks the feasibility of the conditions in a particular situational problem. The parallelization is based on computational threads organized in a variant of the program model "pool of threads" [4]. In this variant, the threads are synchronized with each other while they receive situational problems to solve (race condition). This excludes the solution of one situational problem by more than one thread. The threads are also synchronized with the parent thread, which initiates the calculations and waits for them to complete at each step of the simulated time. Each thread takes one or more situational problems for processing, according to the specified value of the parameter known as granularity.

Situational problems designer. A dialogue editor of situational problems is developed as an auxiliary tool. The compilation of a situational problem is initiated, situational conditions are successively selected, and their respective parameters and restrictions are set with the help of dialogue controls. An already compiled situational problem can be rejected or approved and saved as a template for future use. One already assembled situational problem can be related to a particular satellite or all satellites of the respective multi-satellite system.

Examples of applications. The investigation of the geometric properties of the Earth's shadow concerning satellite orbits with different parameters is not of heuristic interest but is of practical importance. In general, the ratio between the illuminated and dark parts of an orbit depends on the mutual spatial orientation of the shadow and the orbital plane. When orbits are elliptical, their orientation in the orbital plane is also important.

Two cases of application of the considered situational conditions will be shown. The first one is related to a simulation of the conversion of solar energy into electrical. The simulation is based on two situational problems, each with one condition for the lit and dark parts of the orbit, respectively. The second case involves observations of the Earth's surface. Two situational problems are solved again, each with two conditions this time. One of them, common to both problems, is related to the passage of the satellite above a certain region. The other two conditions (each for one of the problems, respectively) require the satellite to be either in the dark part of the trajectory or the lit one.

Conversion of solar energy into electrical in the lit part of the trajectory. The conversion of solar energy into electrical based on solar panels, the charging of batteries and the satellite service subsystems and scientific instruments' energy consumption are objects of computer simulations at different stages of mission preparation.

The amount of converted solar energy on the sunlit segment of the satellite orbit, depending on geometric conditions, is related to Earth's shadow orientation (Sun's position on the ecliptic) and inclination of the orbit. The computer simulations are based on a model of a satellite's circular orbit with a radius of 7×10^3 km. The Sun's position is close to the autumnal equinox. The numerical integration of the satellite motion is performed by a step of 10 seconds. The passage through the penumbra here is ignored due to the insignificant energy contribution in low Earth orbits. Two cases for the longitude of the ascending node $\Omega_1 = 30^\circ$ and $\Omega_2 = 60^\circ$ with different values of eccentricity i are considered (Fig. 2).

The energy has a linear increase in the illuminated part of the orbit and remains constant in the Earth's shadow. The time spent in the umbra (or the lit zone, respectively) depends on both the length of the ascending node and the inclination.

Henceforth, the conversion of solar energy into electrical, its accumulation and consumption by the various satellite subsystems for space mission purposes may be the subject of another analysis.

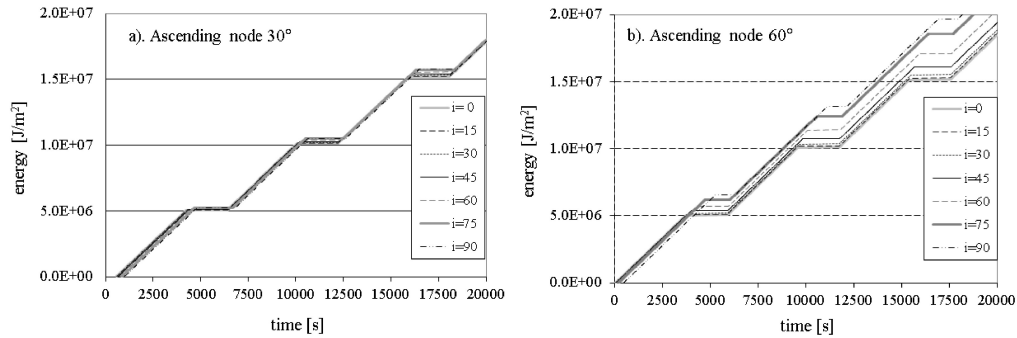


Fig. 2. An accumulation of the solar energy per unit area of the illuminated part of the satellite's orbit depending on the inclination and the longitude of the ascending node

Passing above a circular region of the Earth's surface at the light and dark parts of the trajectory. Here we consider a simplified case of two situational problems each with two conditions. Common to both conditions is that the satellite is located above a circular segment of the Earth's surface with coordinates of the centre ($\varphi = 0^\circ$, $\lambda = 0^\circ$) and angular radius $\delta = 10^\circ$. As a second condition, one problem checks whether the satellite is in the shadow of the Earth, and the other – whether it is in the lit zone. Actually, additional conditions are possible. The inclination of the satellite's orbit i is zero to make the results of the analysis presented in Fig. 3 easier. For the same reason, the centre of the circular region is at the Earth's equator (λ coordinate does not matter) and the

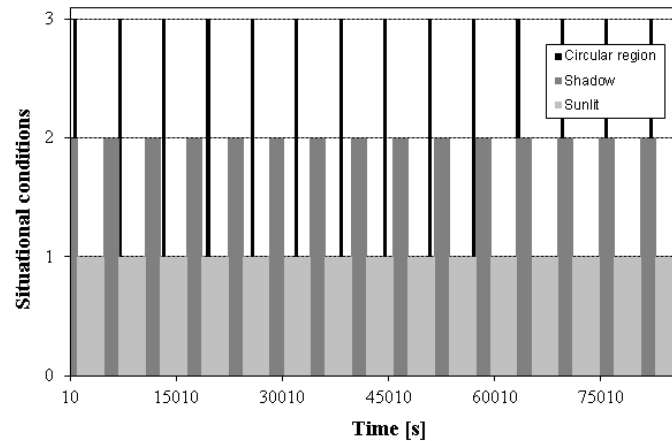


Fig. 3. “Circular region” presents the moments when the satellite is over a circular segment; “Shadow” and “Sunlit” present moments when the satellite is on the dark or sunlit part of the orbit, respectively

angular radius δ is 10 degrees. The values 1, 2 and 3 are selected to represent the time diagrams of the situational conditions on the graph.

The number of passages of a satellite above any arbitrary region on the Earth's surface depends on its size, geographical location and the geometry of the satellite orbit. These passages are separated in time due to the rotation of the Earth when the satellite orbit is inclined to the equator.

Conclusion and outlook. Situational conditions related to the models of the Earth's shadow and the lit zone are examples of conditions participating in several different situational problems. Such situational problems arise from numerous scientific problems and involve the use of data from diverse instruments located on satellites either of one or more space missions. In this article, attention is directed to the use of these models and situational analysis solver in the situational analysis of space missions.

Performing an analysis related to solving a large number of situational problems, each with several conditions can be time-consuming. Apart from the computational models' optimization of the situational conditions and the application of parallel calculations, the search and research for optimization methods of the situational analysis are an interesting challenge and necessary continuation of the work. In situational problems with more conditions, such possibilities are outlined in several ways. One of them is related to the rules of testing situational conditions. Another possible way to optimize situational analysis is related to solving situational problems with equal situational conditions.

Acknowledgements. The author would like to thank Corresponding Member Peter Getsov for the help in the submission of this paper, and to Ms. R. Dimitrova for the technical support in preparation of the article.

REFERENCES

- [1] POGHOSYAN A., I. LLUCH, H. MATEVOSYAN, A. LAMB, C. A. MORENO et al. (2016) Unified classification for distributed satellite systems. In: 4th International Federated and Fractionated Satellite Systems Workshop, Rome, Italy.
- [2] SELVA D., A. GOLKAR, O. KOROBova, I. L. I. CRUZ, P. COLLOPY et al. (2017) Distributed earth satellite systems: What is needed to move forward? *Journal of Aerospace Information Systems*, **14**(8), 412–438.
- [3] WERTZ J. R., W. J. LARSON (1999) *Space Mission Analysis and Design*, Microcosm Press, Kluwer Academic Publishers, 3rd edn.
- [4] ATANASSOV A. M. (2016) Parallel satellite orbital situational problems solver for space missions design and control, *Advances in Space Research*, **58**(9), 1819–1826.
- [5] CUNNINGHAM F. G. (1962) Calculation of the eclipse factor for elliptical satellite orbits, National Aeronautics and Space Administration.
- [6] EREMENKO R. P. (1965) Exact solution of the shadow equation, *Bulletin of the Institute of Theoretical Astronomy*, **6**(119), (in Russian).

- [7] MULLINS L. D. (1991) Calculating satellite umbra/penumbra entry and exit positions and times, *Journal of the Astronautical Sciences*, **39**(4), 411-422.
- [8] NETA B., D. VALLADO (1998) On Satellite Umbra/Penumbra Entry and Exit Positions, *Journal of the Astronautical Sciences*, **46**(1), 91-104.
- [9] ESCOBAL P. (1965) *Methods of orbit determination*, John Wiley and Sons.
- [10] YANG Y., M. XU, D. WANG, Y. WANG (2016) Towards energy-efficient routing in satellite networks, *IEEE Journal on Selected Areas in Communications*, **34**(12), 3869-3886.
- [11] IVANUSHKIN M. A., V. V. SALMIN, S. I. TKACHENKO, I. S. TKACHENKO, S. S. VOLGIN (2018) Development of methods for ensuring energy balance for the operation of mission equipment of "AIST"-series small satellites under conditions of power limitations, *Samara National Research University, Aerospace and Mechanical Engineering*, **17**(4), 57-66. DOI: 10.18287/2541-7533-2018-17-4-57-66.
- [12] KURENKOV V. I. (2020) *Fundamentals of designing spacecraft for optical-electronic observation of the Earth's surface. Calculation of the main characteristics and formation of the design image*, Samara University Press, (in Russian).
- [13] LI Z., M. ZIEBART, S. BHATTARAI, D. HARRISON (2019) A shadow function model based on perspective projection and atmospheric effect for satellites in eclipse, *Advances in Space Research*, **63**(3), 1347-1359.
- [14] ATANASSOV A. M. Developing Situational Conditions and Program Codes for Parallel Situational Analysis Solver based on Conic Umbra/Sunlit Models, *Aerospace Research in Bulgaria*, v.36, accepted.
- [15] RAUBER T., G. RÜNGER (2010) *Parallel Programming: For Multicore and Cluster Systems*, Springer, DOI 10.1007/978-3-642-04818-0.

Space Research and Technology Institute
Stara Zagora Branch
Bulgarian Academy of Sciences
Akad. G. Bonchev St, Bl. 1
1113 Sofia, Bulgaria
e-mail: At_M_Atanassov@yahoo.com