

NUMERICAL SIMULATION OF CUBIC-QUARTIC OPTICAL
SOLITON PERTURBATION BY THE LAPLACE–ADOMIAN
DECOMPOSITION

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Abstract

The current paper is on the numerical analysis of bright and dark cubic-quartic solitons that emerges from the Fokas–Lenells equation with Hamiltonian perturbation terms. The Kerr law of nonlinearity is the source of self-phase modulation. The adopted numerical scheme is the Laplace–Adomian decomposition.

Key words: Fokas–Lenells equation, solitons, cubic-quartic, perturbation, Laplace–Adomian decomposition

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1. Introduction. One of the several models that has attracted attention in the soliton transmission dynamics through monomode optical fibres is the Fokas–Lenells equation (FLE). This model emerged when the key ingredient, chromatic dispersion (CD), runs low that would lead to loss of the delicate balance between dispersion and nonlinearity leading to possible pulse collapse. To offset this, FLE incorporated a nonlinear dispersion term that would replenish this loss.

Lately, the concept of cubic-quartic solitons surfaced that is another measure to counter the loss of necessary delicate balance for the solitons to sustain. In this case, CD is completely eliminated and is being replaced collectively by

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third-order dispersion (3OD) and fourth-order dispersion (4OD) effects. This is what constitutes the cubic-quartic (CQ) optical solitons that is applicable to all models that describe the propagation dynamics of solitons via optical fibres across intercontinental distances.

CQ solitons with FLE have been studied to recover a few analytical results. However, the current paper is the first one to address CQ solitons with FLE purely from a numerical standpoint. The algorithm that has been implemented is the Laplace–Adomian method (LADM). In this context, both bright and dark solitons dynamics have been considered numerically. The surface plots and the error measures are all displayed after the approach has been introduced. The results are displayed in the subsequent sections.

2. The cubic-quartic Fokas–Lenells equation with perturbation terms. The dimensionless formulation of a perturbed CQ-FLE with chromatic dispersion in polarization preserving is represented by

$$(1) \quad iu_t + ia_1u_{xxx} + a_2u_{xxxx} + |u|^2(bu + i\sigma u_x) = i[\alpha u_x + \lambda(|u|^2u)_x + \mu(|u|^2)_xu].$$

This equation was first analyzed in [1–9] and occurs in a variety of systems, including fluid dynamics, solid state physics and condensed matter, nonlinear optics, and plasma physics. The function $u(x, t)$ denotes a complex field envelope in Eq. (1), wherein x and t are, respectively, spatial and temporal variables. Here, a_1 and a_2 denote the third and fourth-order dispersion coefficients, respectively. In addition, self-phase modulation is represented by the coefficient b and σ is a nonlinear term that denotes dispersion. In the perturbative part of Eq. (1), the term appearing in first place denotes intermodal dispersion, while the one in second place indicates self-steepening, and final term represents another type of nonlinear dispersion.

2.1. The solutions involving bright optical solitons. The form of the solution to Eq. (1) for bright optical solitons is provided in [4] by:

$$(2) \quad u(x, t) = A \operatorname{sech}^2[B(x - \nu t)]e^{i[-\kappa x + \omega t + \theta]}.$$

The soliton velocity is denoted by ν , the soliton frequency by ω , the angular velocity by κ , and the phase centre by θ .

Inside this framework, the soliton's amplitude A is given by

$$(3) \quad A = \frac{3a_1\kappa - 6a_2\kappa^2}{10a_2} \sqrt{-\frac{30a_2}{b + \sigma\kappa - \lambda\kappa}}.$$

The inverse width B is given by

$$(4) \quad B = \frac{1}{2} \sqrt{-\frac{3a_1\kappa - 6a_2\kappa^2}{5a_2}},$$

and the relationship between the coefficients of Eq. (1) and the soliton speed is

$$(5) \quad \nu = 4a_2\kappa^3 - 3a_1\kappa^2 - \alpha,$$

the soliton frequency ω is also related to the coefficients of the model to be studied by means of the relationship

$$(6) \quad \omega = -\frac{\kappa(36\kappa a_1^2 - 119a_1 a_2 \kappa^2 + 119a_2 \kappa^2 + 25a_2 \alpha)}{25a_2}$$

and the restrictions on the parameters are as follows

$$(7) \quad \sigma - 3\lambda - 2\mu = 0, \quad a_1 = 4\kappa a_2,$$

$$(8) \quad a_2(b + \sigma\kappa - \lambda\kappa) < 0, \quad a_2(a_1\kappa - 2a_2\kappa^2) < 0$$

κ is any real constant satisfying Eqs. (7) and (8).

2.2. The solutions involving dark optical solitons. The solution to Eq. (1) for dark optical solitons is provided in [5] by:

$$(9) \quad u(x, t) = A \left\{ B + 2 \tanh^2[(x - \nu t)] \right\} e^{i[-\kappa x + \omega t + \theta]}.$$

The soliton velocity is denoted by ν , the soliton frequency by ω , the angular velocity by κ , and the phase centre by θ .

The parameters A and B are expressed in terms of the coefficients of Eq. (1) as follows:

$$(10) \quad A = \sqrt{-\frac{30a_2}{\sigma\kappa - \lambda\kappa + b}}, \quad B = \frac{3\kappa^2 - 20}{15},$$

and the relationship between the coefficients of Eq. (1) and the soliton speed is

$$(11) \quad \nu = \alpha + 8a_2\kappa^3,$$

the soliton frequency ω is also related to the coefficients of the model to be studied by means of the relationship

$$(12) \quad \omega = \frac{16}{3}a_2 - 22a_2\kappa^2 - \alpha\kappa$$

and the restrictions on the parameters are as follows

$$(13) \quad 9\kappa^2 - 30\kappa^2 - 40 = 0, \quad \sigma - 3\lambda - 2\mu = 0, \quad a_1 = 4\kappa a_2,$$

$$(14) \quad a_2(b + \sigma\kappa - \lambda\kappa) < 0$$

κ is any real constant satisfying Eqs. (13) and (14).

3. A summary of the Laplace–Adomian decomposition technique.

ADOMIAN [10] developed the Adomian decomposition approach in the 1980's and applied it to a wide range of stochastic and deterministic issues in science and engineering. It is based on the order to find the solution in the form of a series, as well as the decomposition of the nonlinear operator into a series whose terms are computed recursively using the well-known Adomian polynomials. Subsequently, the concept of combining the decomposition technique with the fundamental Laplace transform, so developing the Adomian–Laplace decomposition method, first appeared [11].

To explain the essential aspects of the Laplace–Adomian decomposition technique, we start by looking at the standard form of a partial differential equation with nonlinear terms, represented operationally by

$$(15) \quad F(u(x, t)) = 0, \text{ with relation to the initial condition } u(x, 0) = f(x).$$

The symbol F represents a differential operator. Now let us disintegrate the operator F into its constituent parts $F = D + R + N$, the expression $D(u) = \frac{\partial u}{\partial t}$ denotes a linear differential operator. The remaining linear and nonlinear components are denoted by the operators R and N , respectively. With these issues, Eq. (15) may be rewritten as

$$(16) \quad Du(x, t) = Ru(x, t) + Nu(x, t).$$

Solving for $Du(x, t)$ and then using the well-known Laplace transform to Eq. (16), results in

$$(17) \quad \mathcal{L}\{Du(x, t)\} = \mathcal{L}\{Ru(x, t) + Nu(x, t)\}.$$

Thus, Eq. (17) is equivalent to

$$(18) \quad su(x, s) = u(x, 0) + \mathcal{L}\{Ru(x, t) + Nu(x, t)\}.$$

By using initial condition, one obtains

$$(19) \quad u(x, s) = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}\{Ru(x, t) + Nu(x, t)\}.$$

Applying now the inverse operator $\mathcal{L}^{-1}$ of $\mathcal{L}$, to each sides of Eq. (19) produces the following result:

$$(20) \quad u(x, t) = f(x) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L}\{Ru(x, t) + Nu(x, t)\} \right].$$

The Laplace–Adomian decomposition approach is based on the notion that the solution $u(x, t)$ may be expressed as a function series.

$$(21) \quad u(x, t) = \sum_{n=0}^{\infty} u_n(x, t).$$

Additionally, the nonlinear term N decomposes as a result of the Adomian technique as

$$(22) \quad Nu(x, t) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n).$$

Each A_n is an Adomian polynomial involving variables $u_0, u_1, \dots, u_n$ that may be computed for every type of nonlinearity in compliance with the recurrent formula [12–14]:

$$(23) \quad \begin{cases} A_0 = N(u_0) & n = 0, \\ A_n = \frac{1}{n} \sum_{k=0}^{n-1} (k+1) u_{k+1} \frac{\partial}{\partial u_0} A_{n-1-k} & n \geq 1. \end{cases}$$

Therefore Adomian's polynomials are given by

$$\begin{aligned} A_0 &= N(u_0) \\ A_1 &= \frac{1}{n} \sum_{k=0}^0 (k+1) u_{k+1} \frac{\partial}{\partial u_0} A_{n-1-k} = u_1 N'(u_0) \\ A_2 &= \frac{1}{n} \sum_{k=0}^1 (k+1) u_{k+1} \frac{\partial}{\partial u_0} A_{n-1-k} = u_2 N'(u_0) + \frac{1}{2} u_1^2 N''(u_0) \\ A_3 &= \frac{1}{n} \sum_{k=0}^2 (k+1) u_{k+1} \frac{\partial}{\partial u_0} A_{n-1-k} = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!} u_1^3 N^{(3)}(u_0) \\ A_4 &= \frac{1}{n} \sum_{k=0}^3 (k+1) u_{k+1} \frac{\partial}{\partial u_0} A_{n-1-k} = u_4 N'(u_0) + \left(\frac{1}{2} u_2^2 + u_1 u_3 \right) N''(u_0) \\ &\quad + \frac{1}{2!} u_1^2 u_2 N^{(3)}(u_0) + \frac{1}{4!} u_1^4 N^{(4)}(u_0) \\ &\vdots \end{aligned}$$

All other polynomials are computed in the same manner.

The formula (23) has the advantage that it does not require tedious calculations involving differentiation with respect to all the variables since only for its application it is necessary to do arithmetic sums and differentiate only with respect to u_0 .

Substituting (21) and (22) into Eq. (20) gives rise to

$$(24) \quad \sum_{n=0}^{\infty} u_n(x, t) = f(x) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \left\{ R \sum_{n=0}^{\infty} u_n(x, t) + \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n) \right\} \right].$$

As a result, Eq. (24) suggests an iterative procedure given by:

$$(25) \quad \begin{cases} u_0(x, t) = f(x), \\ u_{n+1}(x, t) = \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ Ru_n(x, t) + A_n(u_0, u_1, \dots, u_n) \} \right], \quad n = 0, 1, 2, \dots \end{cases}$$

Collecting each of the u_n produced by way of (25) allows us to approximate the solution of Eq. (1) to N -summands by means of

$$(26) \quad u_N(x, t) = \sum_{n=0}^{N-1} u_n(x, t), \quad N \geq 1.$$

This study demonstrated that, using the Adomian decomposition technique in conjunction with the Laplace transform involves significantly less computational work than using the classic Adomian decomposition approach. The suggested approach significantly reduces the amount of computations required. Additionally, the Adomian decomposition process is simple to implement and does not involve linearization or discretization.

4. Cubic-quartic Fokas–Lenells equation solution algorithm derived from the proposed method. In this section, we will illustrate the applicability of LADM to find any particular solution to Eq. (1) that satisfies the condition at $t = 0$: $u(x, 0) = f(x)$.

To apply the LADM method to Eq. (1), we will formulate it in terms of operators as

$$(27) \quad Du(x, t) + Ru(x, t) + N_1u(x, t) + N_2u(x, t) + N_3u(x, t) = 0.$$

In Eq. (27), N_1 , N_2 , and N_3 represent the nonlinear operators acting on u : $-i|u|^2(bu + i\sigma u_x)$, $-\lambda(|u|^2u)_x$, and $-\mu u(|u|^2)_x$, respectively. While $Ru = a_1u_{xxx} - ia_2u_{xxx} - \alpha u_x$ represents the linear differential operator, $Du = u_t$ simply signifies time derivative.

The LADM expresses solution u as an infinite series whose summands are functions, and it is represented as follows:

$$(28) \quad u(x, t) = \sum_{n=0}^{\infty} u_n(x, t).$$

N_1 , N_2 , and N_3 are nonlinear terms operating on the function u and can be decomposed into an infinite sequence of Adomian polynomials denoted by the expressions:

$$(29) \quad N_1u = -i|u|^2(bu + i\sigma u_x) = \sum_{n=0}^{\infty} P_n(u_0, u_1, \dots, u_n),$$

$$(30) \quad N_2 u = -\lambda(|u|^2 u)_x = \sum_{n=0}^{\infty} Q_n(u_0, u_1, \dots, u_n),$$

and

$$(31) \quad N_3 u = -\mu u(|u|^2)_x = \sum_{n=0}^{\infty} S_n(u_0, u_1, \dots, u_n).$$

In expressions (29)–(31), P_n , Q_n , and R_n are the polynomials known as Adomian polynomials, which may be determined using the method provided by Eq. (23), i.e.

$$P_0 = N_1(u_0), \quad Q_0 = N_2(u_0), \quad S_0 = N_3(u_0),$$

and for every $n \geq 1$ we have

$$(32) \quad \begin{aligned} P_n &= \frac{1}{n} \sum_{k=0}^{n-1} (k+1) u_{k+1} \frac{\partial}{\partial u_0} P_{n-1-k}, \\ Q_n &= \frac{1}{n} \sum_{k=0}^{n-1} (k+1) u_{k+1} \frac{\partial}{\partial u_0} Q_{n-1-k}, \\ S_n &= \frac{1}{n} \sum_{k=0}^{n-1} (k+1) u_{k+1} \frac{\partial}{\partial u_0} S_{n-1-k}. \end{aligned}$$

Calculating obtains the first Adomian polynomials, which are

$$\begin{aligned} P_0 &= -i b u_0^2 \bar{u}_0, \\ P_1 &= -2 i b u_0 u_1 \bar{u}_0 - i b u_0^2 \bar{u}_1, \\ P_2 &= -2 i b u_0 u_2 \bar{u}_0 - i b u_1^2 \bar{u}_0 - 2 i b u_0 u_1 \bar{u}_1 - i b u_0^2 \bar{u}_2, \\ P_3 &= -2 i b u_0 u_3 \bar{u}_0 - 2 i b u_1 u_2 \bar{u}_0 - 2 i b u_0 u_2 \bar{u}_1 - i b u_1^2 \bar{u}_1 \\ &\quad - 2 i b u_0 u_1 \bar{u}_2 - i b u_0^2 \bar{u}_3, \\ P_4 &= -i b \bar{u}_0 u_2^2 - 2 i b u_0 \bar{u}_0 u_4 - 2 i b \bar{u}_0 u_1 u_3 - 2 i b u_0 \bar{u}_1 u_3 \\ &\quad - 2 i b u_1 \bar{u}_1 u_2 + 2 u_0 \bar{u}_2 u_2 - i b u_1^2 \bar{u}_2 - 2 i b u_0 \bar{u}_1 u_3 \\ &\quad - i b u_0^2 \bar{u}_4, \\ Q_0 &= -(\lambda + \mu) u_0^2 \bar{u}_{0x}, \\ Q_1 &= -(\lambda + \mu) (u_0^2 \bar{u}_{1x} + 2 u_0 u_1 \bar{u}_{0x}), \\ Q_2 &= -(\lambda + \mu) (u_1^2 \bar{u}_{0x} + u_0^2 \bar{u}_{2x} + 2 u_0 u_1 \bar{u}_{1x} + 2 u_0 u_2 \bar{u}_{0x}), \\ Q_3 &= -(\lambda + \mu) (u_1^2 \bar{u}_{1x} + u_0^2 \bar{u}_{3x} + 2 u_0 u_1 \bar{u}_{2x} + 2 u_0 u_2 \bar{u}_{1x} \\ &\quad + 2 u_0 u_3 \bar{u}_{0x} + 2 u_1 u_2 \bar{u}_{0x}), \\ Q_4 &= -(\lambda + \mu) (u_2^2 \bar{u}_{0x} + u_1^2 \bar{u}_{2x} + 2 u_0 u_1 \bar{u}_{3x} + 2 u_0 u_2 \bar{u}_{2x} \\ &\quad + 2 u_0 u_3 \bar{u}_{1x} + 2 u_0 u_4 \bar{u}_{0x} + 2 u_1 u_2 \bar{u}_{1x} + 2 u_1 u_3 \bar{u}_{0x}), \end{aligned}$$

$$\begin{aligned}
S_0 &= (\sigma - 2\lambda - \mu)u_0\bar{u}_0u_{0x}, \\
S_1 &= (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{1x} + u_0\bar{u}_1u_{0x} + u_1\bar{u}_0u_{0x}), \\
S_2 &= (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{2x} + u_0\bar{u}_1u_{1x} + u_0\bar{u}_2u_{0x} + u_1\bar{u}_0u_{1x} \\
&\quad + u_1\bar{u}_1u_{0x} + u_2\bar{u}_0u_{0x}), \\
S_3 &= (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{3x} + u_0\bar{u}_1u_{2x} + u_0\bar{u}_2u_{1x} + u_0\bar{u}_3u_{0x} + u_1\bar{u}_0u_{2x} \\
&\quad + u_1\bar{u}_1u_{1x} + u_1\bar{u}_2u_{0x} + u_2\bar{u}_0u_{1x} + u_2\bar{u}_1u_{0x} + u_3\bar{u}_0u_{0x}), \\
S_4 &= (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{4x} + u_0\bar{u}_1u_{3x} + u_0\bar{u}_2u_{2x} + u_0\bar{u}_3u_{1x} + u_0\bar{u}_4u_{0x} \\
&\quad + u_1\bar{u}_0u_{3x} + u_1\bar{u}_1u_{2x} + u_1\bar{u}_2u_{1x} + u_1\bar{u}_3u_{0x} + u_2\bar{u}_0u_{2x} + u_2\bar{u}_1u_{1x} \\
&\quad + u_2\bar{u}_2u_{0x} + u_3\bar{u}_0u_{1x} + u_3\bar{u}_1u_{0x} + u_4\bar{u}_0u_{0x}),
\end{aligned}$$

Then, the first Adomian polynomials that correspond to the nonlinear component N_1, N_2, N_3 are added to obtain

$$\begin{aligned}
A_0 &= -ibu_0^2\bar{u}_0 - (\lambda + \mu)u_0^2\bar{u}_{0x} + (\sigma - 2\lambda - \mu)u_0\bar{u}_0u_{0x}, \\
A_1 &= -2ibu_0u_1\bar{u}_0 - ibu_0^2\bar{u}_1 - (\lambda + \mu)(u_0^2\bar{u}_{1x} + 2u_0u_1\bar{u}_{0x}) \\
&\quad + (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{1x} + u_0\bar{u}_1u_{0x} + u_1\bar{u}_0u_{0x}), \\
A_2 &= -2ibu_0u_2\bar{u}_0 - ibu_1^2\bar{u}_0 - 2ibu_0u_1\bar{u}_1 - ibu_0^2\bar{u}_2 \\
&\quad - (\lambda + \mu)(u_1^2\bar{u}_{0x} + u_0^2\bar{u}_{2x} + 2u_0u_1\bar{u}_{1x} + 2u_0u_2\bar{u}_{0x}) \\
&\quad + (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{2x} + u_0\bar{u}_1u_{1x} + u_0\bar{u}_2u_{0x} + u_1\bar{u}_0u_{1x} \\
&\quad + u_1\bar{u}_1u_{0x} + u_2\bar{u}_0u_{0x}), \\
A_3 &= -2ibu_0u_3\bar{u}_0 - 2ibu_1u_2\bar{u}_0 - 2ibu_0u_2\bar{u}_1 - ibu_1^2\bar{u}_1 - 2ibu_0u_1\bar{u}_2 \\
&\quad - ibu_0^2\bar{u}_3 - (\lambda + \mu)(u_1^2\bar{u}_{1x} + u_0^2\bar{u}_{3x} + 2u_0u_1\bar{u}_{2x} + 2u_0u_2\bar{u}_{1x} \\
&\quad + 2u_0u_3\bar{u}_{0x} + 2u_1u_2\bar{u}_{0x}) + (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{3x} + u_0\bar{u}_1u_{2x} \\
&\quad + u_0\bar{u}_2u_{1x} + u_0\bar{u}_3u_{0x} + u_1\bar{u}_0u_{2x} + u_1\bar{u}_1u_{1x} + u_1\bar{u}_2u_{0x} \\
&\quad + u_2\bar{u}_0u_{1x} + u_2\bar{u}_1u_{0x} + u_3\bar{u}_0u_{0x}).
\end{aligned}$$

Similarly, the remainder of the polynomials are produced in a similar way.

By implementing the Laplace integral transform with respect to t on both parts of Eq. (27) and using its linearity, the following result is obtained:

$$(33) \quad \mathcal{L}\{Du(x, t)\} = -\mathcal{L}\{Ru(x, t)\} - \mathcal{L}\{N_1u(x, t)\} - \mathcal{L}\{N_2u(x, t)\} - \mathcal{L}\{N_3u(x, t)\}.$$

Due to the differentiation characteristic of the Laplace transform, (33) may be expressed as

$$\begin{aligned}
(34) \quad s\mathcal{L}\{u(x, t)\} - u(x, 0) &= -\mathcal{L}\{Ru(x, t)\} - \mathcal{L}\{N_1u(x, t)\} - \mathcal{L}\{N_2u(x, t)\} \\
&\quad - \mathcal{L}\{N_3u(x, t)\}.
\end{aligned}$$

Rewriting the preceding expression,

$$\begin{aligned}
(35) \quad \mathcal{L}\{u(x, t)\} &= \frac{1}{s}u(x, 0) - \frac{1}{s}(\mathcal{L}\{Ru(x, t)\} + \mathcal{L}\{N_1u(x, t)\} + \mathcal{L}\{N_2u(x, t)\} \\
&\quad + \mathcal{L}\{N_3u(x, t)\}).
\end{aligned}$$

By substituting (28), (29), (30) and (31) into (35), we acquire

$$(36) \quad \mathcal{L} \left\{ \sum_{n=0}^{\infty} u_n(x, t) \right\} = \frac{f(x)}{s} - \frac{1}{s} \left(\mathcal{L} \left\{ R \sum_{n=0}^{\infty} u_n(x, t) \right\} + \mathcal{L} \left\{ \sum_{n=0}^{\infty} P_n \right\} \right. \\ \left. + \mathcal{L} \left\{ \sum_{n=0}^{\infty} Q_n \right\} + \mathcal{L} \left\{ \sum_{n=0}^{\infty} S_n \right\} \right).$$

Comparing algebraically both sides of Eq. (36), the following relationships emerge:

$$(37) \quad \mathcal{L}\{u_0(x, t)\} = \frac{f(x)}{s}$$

in general, we deduct recursively:

$$(38) \quad \mathcal{L}\{u_{n+1}(x, t)\} = -\frac{1}{s} (\mathcal{L}\{Ru_n(x, t)\} + \mathcal{L}\{P_n + Q_n + S_n\}), \quad n \geq 1.$$

The following recurrence formulae are deduced for each $n = 0, 1, 2, \dots$, by employing the inverse Laplace transformation

$$(39) \quad \begin{cases} u_0(x, t) = f(x), \\ u_{n+1}(x, t) = -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ Ru_n(x, t) + (P_n + Q_n + S_n)(u_0, \dots, u_n) \} \right]. \end{cases}$$

5. Illustrating examples and graphical solutions. This section will run numerical simulations of both bright and dark solitons using the technique presented in the previous section.

5.1. Simulations of bright optical solitons. For our numerical simulations, we will consider the parameters shown in Table 1. Figure 1 shows the soliton evolution profile for one of the examples.

T a b l e 1

Bright optical solitons

Examples	a_1	a_2	b	σ	α	λ	μ	N	Max Error
1	2.20	-0.35	4.90	0.01	0.08	-1.40	1.02	12	1.20×10^{-9}
2	0.55	0.16	0.90	3.80	0.40	2.05	1.33	12	2.22×10^{-9}

5.2. Simulations of dark optical solitons. For our numerical simulations, we will consider the parameters shown in Table 2. Figure 2 shows the soliton evolution profile for one of the examples.

6. Conclusions. The current paper is the analysis from numerical perspective for CQ solitons that comes with Hamiltonian perturbations which did not destroy the integrability of the model. The LADM approach yielded bright and dark CQ solitons that emerged from the FLE. The error plots, surface plots as

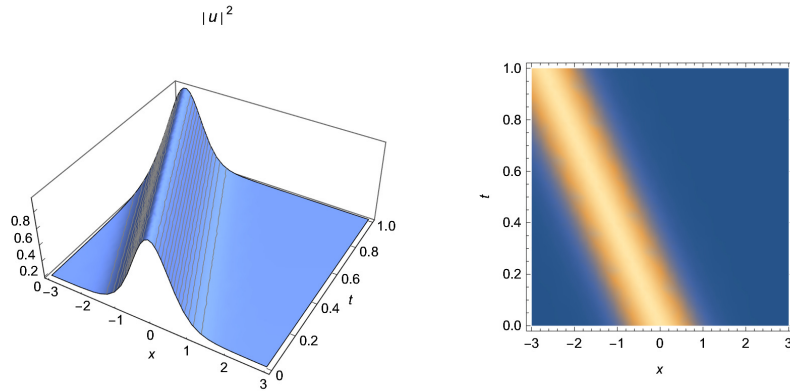


Fig. 1. Bright solitons: $|u|^2$ plot profile obtained by LADM and density diagram showing the produced wave amplitude for the parameter values used in example 1

T a b l e 2
Dark optical solitons

Examples	a_1	a_2	b	σ	α	λ	μ	N	Max Error
3	0.87	-0.34	1.23	0.86	2.02	0.06	0.44	12	1.00×10^{-9}
4	0.60	-0.88	2.22	1.01	3.20	-0.90	1.17	12	1.45×10^{-9}

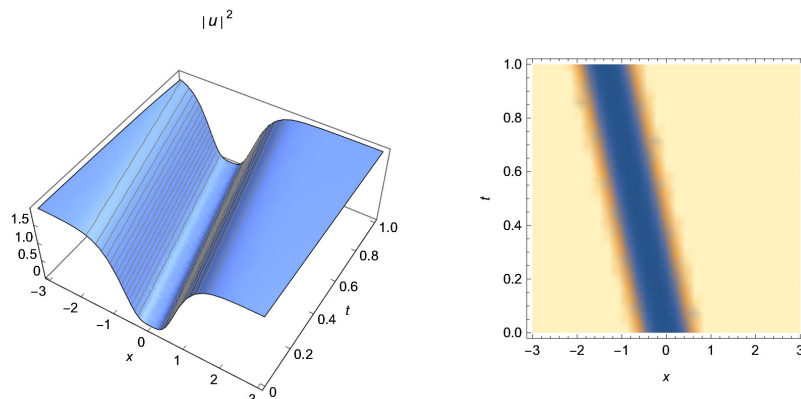


Fig. 2. Dark solitons: $|u|^2$ plot profile obtained by LADM and density diagram showing the produced wave amplitude for the parameter values used in example 3

well as the contour plots of bright and dark CQ solitons are exhibited in the work. To move on, in future this model will be addressed on a generalized setting. While the current model is with cubic nonlinearity, the subsequent paper would be with power law of nonlinearity whose results would be disseminated in time. These results are currently in the works.

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