

DISPERSIVE OPTICAL SOLITONS
WITH RADHAKRISHNAN–KUNDU–LAKSHMANAN
EQUATION HAVING MULTIPLICATIVE WHITE NOISE
BY ENHANCED KUDRYASHOV'S METHOD
AND EXTENDED SIMPLEST EQUATION

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Abstract

This article is about dispersive optical solitons with Radhakrishnan–Kundu–Lakshmanan equation with power-law of nonlinear refractive index in presence of multiplicative white noise by the usage of Itô calculus. The soliton solutions contain the Wiener process embedded into its phase component. Two integration algorithms make this retrieval possible and they are enhanced Kudryashov's method and the extended simplest equation approach.

Key words: solitons, white noise, Wiener process

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1. Introduction. The theory of optical solitons has made unbelievable advances in the world of telecommunications. There are several issues that have been newly addressed. These are dispersive solitons, highly-dispersive solitons, pure-cubic solitons, cubic-quartic solitons, dispersion-managed solitons and many more. The current paper focuses on dispersive optical solitons for which there exist several models that are studied. They are Schrödinger–Hirota equation,

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Fokas–Lenells equation and several others. The current paper will address dispersive solitons with one such model that has been around for decades. This is the Radhakrishnan–Kundu–Lakshmanan (RKL) equation [3–9, 17–20]. To keep it on a generalized setting, the model is considered with power-law nonlinearity.

The multiplicative white noise is also incorporated to gain further insight into the dynamics of dispersive soliton transmission. Itô calculus is implemented to analyze the model [1, 2, 10–16]. The enhanced Kudryashov’s method and the extended simplest equation approach are the integration tools that recover the soliton solutions to the model with multiplicative white noise. The results are displayed after jotting down the essential mathematics. The details follow through.

1.1. Governing model. The dimensionless form of RKL equation with power-law of nonlinear refractive index having multiplicative noise in the Itô sense is written, for the first time, as

$$(1) \quad i q_t + a q_{xx} + b |q|^{2n} q + i \left[\beta q_{xxx} + \alpha \left(|q|^{2n} q \right)_x \right] + \sigma q \frac{dW(t)}{dt} = 0,$$

where $q(x, t)$ denotes complex-valued function representing the wave profile, while a, b, β, α and σ are real-valued constants while $i = \sqrt{-1}$. In Eq. (1), the first term is the linear temporal evolution, a is the coefficient of chromatic dispersion, b is the coefficient of power law of self-phase modulation, β is the coefficient of third order dispersion (3OD), α is the coefficient of self-steepening (SS) term for the short pulses, σ is the coefficient of noise strength and $W(t)$ is the standard Wiener process, such that $dW(t)/dt$ is the white noise. The exponent n in Eq. (1) represents the power law nonlinearity parameter. If $\sigma = 0$, Eq. (1) reduces to the familiar RKL equation.

2. Mathematical setup. To disband the model (1), we postulate the following assumption

$$(2) \quad q(x, t) = P(\xi) \exp i \left[\Psi(x, t) + \sigma W(t) - \sigma^2 t \right],$$

where

$$(3) \quad \Psi(x, t) = -\kappa x + \omega t, \quad \xi = x - Vt.$$

Here κ, ω and V are nonzero real-valued constants, while $P(\xi)$ and $\Psi(x, t)$ are real-valued functions. ξ is the wave variable, ω is the frequency, κ is the wave number, and V is the velocity. $\Psi(x, t)$ is the phase component and $P(\xi)$ is the amplitude component. Plugging (2) and (3) into (1) gives the real part

$$(4) \quad (a + 3\beta\kappa) P''(\xi) + (b + \kappa\alpha) P^{2n+1}(\xi) - (\omega - \sigma^2 + a\kappa^2 + \beta\kappa^3) P(\xi) = 0$$

and the imaginary part

$$(5) \quad \beta P'''(\xi) + (2n + 1) \alpha P^{2n}(\xi) P'(\xi) - (V + 2a\kappa + 3\beta\kappa^2) P'(\xi) = 0.$$

Integrating Eq. (5) with zero-integration constant, we arrive at

$$(6) \quad \beta P''(\xi) + \alpha P^{2n+1}(\xi) - (V + 2a\kappa + 3\beta\kappa^2) P(\xi) = 0.$$

Eqs. (4) and (6) have the same form by virtue of the restrictions

$$(7) \quad \frac{a + 3\beta\kappa}{\beta} = \frac{(b + \kappa\alpha)}{\alpha} = \frac{\omega - \sigma^2 + a\kappa^2 + \beta\kappa^3}{V + 2a\kappa + 3\beta\kappa^2},$$

and from which, one reveals the wave number of the soliton

$$(8) \quad \kappa = \frac{b\beta - a\alpha}{2\alpha\beta},$$

and the velocity

$$(9) \quad V = \frac{(\omega - \sigma^2 + a\kappa^2 + \beta\kappa^3) \alpha - (2a\kappa + 3\beta\kappa^2) (b + \kappa\alpha)}{(b + \kappa\alpha)},$$

provided $\alpha\beta \neq 0$ and $b + \kappa\alpha \neq 0$. Now, balancing the terms $P''(\xi)$ and $P^{2n+1}(\xi)$ in Eq. (4) yields the balance number $N = \frac{1}{n}$, $n \geq 1$. Since the balance number is not integer, then we take the following transformation

$$(10) \quad P(\xi) = H^{\frac{1}{n}}(\xi),$$

where $H(\xi)$ is a new function of ξ such that $H(\xi) > 0$. Now, Eq. (4) changes to the new equation

$$(11) \quad (a + 3\beta\kappa) [(1 - n)H'^2(\xi) + nH(\xi)H''(\xi)] + n^2 (b + \kappa\alpha) H^4(\xi) - n^2 (\omega - \sigma^2 + a\kappa^2 + \beta\kappa^3) H^2(\xi) = 0.$$

In the next section, we solve Eq. (11) using the following approaches.

3. Enhanced Kudryashov's approach. Based on this method, assume that Eq. (11) has the formal solution

$$(12) \quad H(\xi) = \sum_{s=0}^M B_s R^s(\xi), \quad B_M \neq 0$$

and the function $R(\xi)$ satisfies the ODE

$$(13) \quad R'^2(\xi) = R^2(\xi) [1 - \chi R^{2p}(\xi)] \ln^2 K, \quad 0 < K \neq 1,$$

where B_s , $s = 0, 1, 2, \dots, M$, and χ are parameters and $p > 0$ is an integer. Balancing $H(\xi)H''(\xi)$ with $H^4(\xi)$ in Eq. (11), we get

$$(14) \quad 2M + 2p = 4M \implies M = p.$$

Let us now discuss the following cases:

Case 1. Choose $p = 1$, then $M = 1$. Now, we have the formal solution

$$(15) \quad H(\xi) = B_0 + B_1 R(\xi), \quad B_1 \neq 0,$$

where B_0 and B_1 are parameters. Here $R(\xi)$ satisfies the differential equation

$$(16) \quad R'^2(\xi) = R^2(\xi) [1 - \chi R^2(\xi)] \ln^2 K, \quad 0 < K \neq 1.$$

Substituting (15), (16) in (11) and setting all coefficients of $[R(\xi)]^{q_1}$, $q_1 = 0, \dots, 4$, $[R'(\xi)]^{q_2}$, $q_2 = 0, 1$, to zero, we have the results

$$(17) \quad n = n, \quad \omega = \frac{(a + 3\beta\kappa) \ln^2 K - (\beta\kappa^3 + a\kappa^2 - \sigma^2) n^2}{n^2},$$

$$B_0 = 0, \quad B_1 = \sqrt{\frac{\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}},$$

provided $\chi(a + 3\beta\kappa)(b + \kappa\alpha) > 0$. By substituting (17) along with the well-known solution of Eq. (16) obtained in [10] into (15), we obtain the solution

$$(18) \quad q(x, t) = \left[\sqrt{\frac{\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \left(\frac{4S}{4S^2 K^\xi + \chi K^{-\xi}} \right) \right]^{\frac{1}{n}} \times e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Solution (18) can be rewritten in the form

$$(19) \quad q(x, t) = \sqrt[2n]{\frac{\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \times \left(\frac{4S}{(4S^2 + \chi) \cosh(\xi \ln K) + (4S^2 - \chi) \sinh(\xi \ln K)} \right)^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

which represents the combo bright-singular soliton solution. In particular, when $\chi = 4S^2$, we have the bright soliton solution

$$(20) \quad q(x, t) = \left[\sqrt{\frac{(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \operatorname{sech}(\xi \ln K) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) > 0$, while if $\chi = -4S^2$, we have the singular soliton solution

$$(21) \quad q(x, t) = \left[\sqrt{-\frac{(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \operatorname{csch}(\xi \ln K) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$.

Case 2. Choose $p = 2$, then $M = 2$. Now, we have the formal solution

$$(22) \quad H(\xi) = B_0 + B_1 R(\xi) + B_2 R^2(\xi), \quad B_2 \neq 0,$$

where B_0 , B_1 and B_2 are parameters. Here $R(\xi)$ satisfies the differential equation

$$(23) \quad R'^2(\xi) = R^2(\xi) [1 - \chi R^4(\xi)] \ln^2 K, \quad 0 < K \neq 1.$$

Substituting (22), (23) in (11) and setting all coefficients of $[R(\xi)]^{q_1}$, $q_1 = 0, \dots, 8$, $[R'(\xi)]^{q_2}$, $q_2 = 0, 1$, to zero, we have the results

$$(24) \quad \begin{aligned} n = n, \quad \omega &= \frac{4(a + 3\beta\kappa) \ln^2 K - (\beta\kappa^3 + a\kappa^2 - \sigma^2)n^2}{n^2}, \\ B_0 &= 0, \quad B_1 = 0, \quad B_2 = \sqrt{\frac{4\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}}, \end{aligned}$$

provided $\chi(a + 3\beta\kappa)(b + \kappa\alpha) > 0$. Substituting (24) along with the well-known solution of Eq. (23), into (22), we have the solution

$$(25) \quad q(x, t) = \left[\sqrt{\frac{4\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \left(\frac{4S}{4S^2 K^{2\xi} + \chi K^{-2\xi}} \right) \right]^{\frac{1}{n}} \times e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Solution (25) can be rewritten in the form

$$(26) \quad q(x, t) = \sqrt[n]{\frac{4\chi(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \times \left(\frac{4S}{(4S^2 + \chi) \cosh(2\xi \ln K) + (4S^2 - \chi) \sinh(2\xi \ln K)} \right)^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

which represents the combo bright-singular soliton solution. In particular, when $\chi = 4S^2$, we have the bright soliton solution

$$(27) \quad q(x, t) = \left[\sqrt{\frac{4(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \operatorname{sech}(2\xi \ln K) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) > 0$, while if $\chi = -4S^2$, we have the singular soliton solution

$$(28) \quad q(x, t) = \left[\sqrt{-\frac{4(n+1)(a + 3\beta\kappa) \ln^2 K}{n^2(b + \kappa\alpha)}} \operatorname{csch}(2\xi \ln K) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$.

Similarly, we can find many other solutions by choosing other values for p and M .

4. The extended simplest equation approach. Eq. (11) has the formal solution:

$$(29) \quad H(\xi) = \chi_0 + \chi_1 \left[\frac{\phi'(\xi)}{\phi(\xi)} \right] + \rho_0 \left[\frac{1}{\phi(\xi)} \right], \quad \chi_1^2 + \rho_0^2 \neq 0,$$

and $\phi(\xi)$ is the solution of equation

$$(30) \quad \phi''(\xi) + \delta\phi(\xi) = v,$$

where δ , v , χ_0 , χ_1 and ρ_0 are constants. Now, we have the following three types:

Type 1. $\delta < 0$. In this case, we substitute (29) into Eq. (11) and use Eq. (30) together with the relation

$$(31) \quad \left(\frac{\phi'(\xi)}{\phi(\xi)} \right)^2 = \Delta_1 \left(\frac{1}{\phi(\xi)} \right)^2 - \delta + \frac{2v}{\phi(\xi)},$$

where $\Delta_1 = \delta (A_1^2 - A_2^2) - v^2/\delta$, while A_1 and A_2 are constants, gives way to the following results.

Result 1.

$$(32) \quad \begin{aligned} n = n, \quad \omega = -\frac{(a + 3\beta\kappa)\delta + (\beta\kappa^3 + a\kappa^2 - \sigma^2) n^2}{n^2}, \quad v = 0, \\ \chi_0 = 0, \quad \chi_1 = 0, \quad \rho_0 = \sqrt{-\frac{(n+1)(a + 3\beta\kappa)(A_1^2 - A_2^2)\delta}{n^2(b + \kappa\alpha)}}, \end{aligned}$$

provided $(a + 3\beta\kappa)(A_1^2 - A_2^2)(b + \kappa\alpha) > 0$.

Consequently, we obtain the following combo bright-singular soliton solution of Eq. (1) as follows

$$(33) \quad q(x, t) = \sqrt[n]{-\frac{(n+1)(a + 3\beta\kappa)(A_1^2 - A_2^2)\delta}{n^2(b + \kappa\alpha)}} \times \left(\frac{1}{A_1 \cosh(\sqrt{-\delta}\xi) + A_2 \sinh(\sqrt{-\delta}\xi)} \right)^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

In particular, if we set $A_1 = 0$, $A_2 \neq 0$ in (33), we have the singular soliton solution

$$(34) \quad q(x, t) = \left[\sqrt{\frac{(n+1)(a + 3\beta\kappa)\delta}{n^2(b + \kappa\alpha)}} \operatorname{csch}(\sqrt{-\delta}\xi) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$, while if we set $A_1 \neq 0$, $A_2 = 0$ in (33), we have the bright soliton solution

$$(35) \quad q(x, t) = \left[\sqrt{-\frac{(n+1)(a + 3\beta\kappa)\delta}{n^2(b + \kappa\alpha)}} \operatorname{sech}(\sqrt{-\delta}\xi) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) > 0$.

Result 2.

$$(36) \quad \begin{aligned} n &= 1, \quad \omega = \frac{1}{2}(a + 3\beta\kappa)\delta - (\beta\kappa^3 + a\kappa^2 - \sigma^2), \\ \chi_0 &= \sqrt{\frac{(a + 3\beta\kappa)\delta}{2(b + \kappa\alpha)}}, \quad \chi_1 = 0, \quad \rho_0 = -\frac{2v}{\delta} \sqrt{\frac{(a + 3\beta\kappa)\delta}{2(b + \kappa\alpha)}}, \quad A_1 = A_2, \end{aligned}$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$.

Consequently, we obtain the following combo bright-singular soliton solution of Eq. (1) as follows

$$(37) \quad q(x, t) = \sqrt{\frac{(a + 3\beta\kappa)\delta}{2(b + \kappa\alpha)}} \left[1 - \frac{2v}{\delta A_1 \left(\cosh(\sqrt{-\delta}\xi) + \sinh(\sqrt{-\delta}\xi) + \frac{v}{\delta A_1} \right)} \right] \times e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Result 3.

$$(38) \quad \begin{aligned} n &= 1, \quad \omega = \frac{1}{2}(a + 3\beta\kappa)\delta - (\beta\kappa^3 + a\kappa^2 - \sigma^2), \\ \chi_0 &= 0, \quad \chi_1 = \sqrt{-\frac{(a + 3\beta\kappa)}{2(b + \kappa\alpha)}}, \quad \rho_0 = \sqrt{-\frac{(a + 3\beta\kappa)\Delta_1}{2(b + \kappa\alpha)}}, \end{aligned}$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$ and $\Delta_1 > 0$.

$$(39) \quad q(x, t) = \sqrt{-\frac{(a + 3\beta\kappa)}{2(b + \kappa\alpha)}} \left[\Omega_1(\xi) + \sqrt{\Delta_1} \Omega_2(\xi) \right] e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

where

$$(40) \quad \begin{aligned} \Omega_1(\xi) &= \frac{A_1 \sinh(\sqrt{-\delta}\xi) + A_2 \cosh(\sqrt{-\delta}\xi)}{A_1 \cosh(\sqrt{-\delta}\xi) + A_2 \sinh(\sqrt{-\delta}\xi) + \frac{v}{\delta}}, \\ \Omega_2(\xi) &= \frac{1}{A_1 \cosh(\sqrt{-\delta}\xi) + A_2 \sinh(\sqrt{-\delta}\xi) + \frac{v}{\delta}}. \end{aligned}$$

In particular, if we set $A_1 = 0$, $A_2 \neq 0$, $v = 0$ in (39), we have the combo-singular soliton solution

$$(41) \quad q(x, t) = \sqrt{-\frac{(a + 3\beta\kappa)}{2(b + \kappa\alpha)}} \left[\coth(\sqrt{-\delta}\xi) + \sqrt{-\delta} \operatorname{csch}(\sqrt{-\delta}\xi) \right] \times e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Type 2. $\delta > 0$. In this case, we substitute (29) into Eq. (11) and use Eq. (30) together with the relation

$$(42) \quad \left(\frac{\phi'(\xi)}{\phi(\xi)} \right)^2 = \Delta_2 \left(\frac{1}{\phi(\xi)} \right)^2 - \delta + \frac{2v}{\phi(\xi)},$$

where $\Delta_2 = \delta (A_1^2 + A_2^2) - v^2/\delta$ gives way to the following results:

Result 1.

$$(43) \quad \begin{aligned} n &= n, \quad \omega = -\frac{(a + 3\beta\kappa)\delta + (\beta\kappa^3 + a\kappa^2 - \sigma^2)n^2}{n^2}, \quad v = 0, \\ \chi_0 &= 0, \quad \chi_1 = 0, \quad \rho_0 = \sqrt{-\frac{(n+1)(a+3\beta\kappa)(A_1^2 + A_2^2)\delta}{n^2(b+\kappa\alpha)}}, \end{aligned}$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$.

Consequently, we obtain the following trigonometric function solution of Eq. (1) as follows:

$$(44) \quad q(x, t) = \sqrt[n]{-\frac{(n+1)(a+3\beta\kappa)(A_1^2 + A_2^2)\delta}{n^2(b+\kappa\alpha)}} \times \left(\frac{1}{A_1 \cos[\sqrt{\delta}(x-ct)] + A_2 \sin[\sqrt{\delta}(x-ct)]} \right)^{\frac{1}{n}} \times e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

In particular, if we set $A_1 = 0$, $A_2 \neq 0$ in (44), we have the periodic wave solution

$$(45) \quad q(x, t) = \left[\sqrt{-\frac{(n+1)(a+3\beta\kappa)\delta}{n^2(b+\kappa\alpha)}} \csc(\sqrt{\delta}\xi) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

while if we set $A_1 \neq 0$, $A_2 = 0$ in (44), we have the periodic wave solution

$$(46) \quad q(x, t) = \left[\sqrt{-\frac{(n+1)(a+3\beta\kappa)\delta}{n^2(b+\kappa\alpha)}} \sec(\sqrt{\delta}\xi) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Result 2.

$$(47) \quad \begin{aligned} n &= 1, \quad \omega = \frac{1}{2}(a + 3\beta\kappa)\delta - (\beta\kappa^3 + a\kappa^2 - \sigma^2), \\ \chi_0 &= 0, \quad \chi_1 = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}}, \quad \rho_0 = \sqrt{-\frac{(a+3\beta\kappa)\Delta_2}{2(b+\kappa\alpha)}}, \end{aligned}$$

provided $(a + 3\beta\kappa)(b + \kappa\alpha) < 0$ and $\Delta_2 > 0$.

Consequently, we obtain the following periodic solution of Eq. (1)

$$(48) \quad q(x, t) = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}} \left[F_1(\xi) + \sqrt{\Delta_2} F_2(\xi) \right] e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

where

$$(49) \quad \begin{aligned} F_1(\xi) &= \frac{-A_1 \sin(\sqrt{\delta}\xi) + A_2 \cos(\sqrt{\delta}\xi)}{A_1 \cos(\sqrt{\delta}\xi) + A_2 \sin(\sqrt{\delta}\xi) + \frac{v}{\delta}}, \\ F_2(\xi) &= \frac{1}{A_1 \cos(\sqrt{\delta}\xi) + A_2 \sin(\sqrt{\delta}\xi) + \frac{v}{\delta}}. \end{aligned}$$

In particular, if we set $A_1 = 0$, $A_2 \neq 0$, $v = 0$ in (48), we have the periodic wave solution

$$(50) \quad q(x, t) = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}} \left[\cot(\sqrt{\delta}\xi) + \sqrt{\delta} \csc(\sqrt{\delta}\xi) \right] e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

while if we set $A_1 \neq 0$, $A_2 = 0$, $v = 0$ in (48), we have the periodic solution

$$(51) \quad q(x, t) = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}} \left[-\tan(\sqrt{\delta}\xi) + \sqrt{\delta} \sec(\sqrt{\delta}\xi) \right] e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Type 3. $\delta = 0$. In this case, we substitute (29) into Eq. (11) and use Eq. (30) together with the relation

$$(52) \quad \left(\frac{\phi'(\xi)}{\phi(\xi)} \right)^2 = \Delta_3 \left(\frac{1}{\phi(\xi)} \right)^2 + \frac{2v}{\phi(\xi)},$$

where $\Delta_3 = A_1^2 - 2vA_2$ gives way to the following results:

Result 1.

$$(53) \quad \begin{aligned} n &= n, \quad \omega = -(\beta\kappa^3 + a\kappa^2 - \sigma^2), \quad v = 0, \\ \chi_0 &= 0, \quad \chi_1 = 0, \quad \rho_0 = \sqrt{-\frac{(n+1)(a+3\beta\kappa)A_1^2}{n^2(b+\kappa\alpha)}}, \end{aligned}$$

provided $(a+3\beta\kappa)(b+\kappa\alpha) < 0$.

Consequently, we obtain the following rational function solution of Eq. (1)

$$(54) \quad q(x, t) = \left[\sqrt{-\frac{(n+1)(a+3\beta\kappa)}{n^2(b+\kappa\alpha)}} \left(\frac{A_1}{A_1\xi + A_2} \right) \right]^{\frac{1}{n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}.$$

Result 2.

$$(55) \quad \begin{aligned} n &= 1, \quad \omega = -(\beta\kappa^3 + a\kappa^2 - \sigma^2), \\ \chi_0 &= 0, \quad \chi_1 = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}}, \quad \rho_0 = \sqrt{-\frac{(a+3\beta\kappa)\Delta_3}{2(b+\kappa\alpha)}}, \end{aligned}$$

provided $(a+3\beta\kappa)(b+\kappa\alpha) < 0$ and $\Delta_3 > 0$.

Consequently, we obtain the following rational function solution of Eq. (1)

$$(56) \quad q(x, t) = \sqrt{-\frac{(a+3\beta\kappa)}{2(b+\kappa\alpha)}} \left[\Pi_1(\xi) + \sqrt{\Delta_3} \Pi_2(\xi) \right] e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]},$$

where

$$(57) \quad \begin{aligned} \Pi_1(\xi) &= \frac{v\xi + A_1}{\frac{v}{2}\xi^2 + A_1\xi + A_2}, \\ \Pi_2(\xi) &= \frac{1}{\frac{v}{2}\xi^2 + A_1\xi + A_2}. \end{aligned}$$

5. Specific values of the power-law parameter. To this aim, we first integrate Eq. (5) to get equation

$$(58) \quad \beta P''(\xi) + \alpha P^{2n+1}(\xi) - (V + 2a\kappa + 3\beta\kappa^2) P(\xi) - c_1 = 0,$$

where c_1 is a non-zero constant of integration. The linearly independent principle on Eq. (4) gives

$$(59) \quad a = -3\beta\kappa, \quad b = -\kappa\alpha, \quad \omega - \sigma^2 + a\kappa^2 + \beta\kappa^3 = 0.$$

5.1. $n = 1/2$. Let us now solve Eq. (58) at $n = 1/2$ taking into account the conditions (59). To this aim, we multiply Eq. (58) by $P'(\xi)$ and integrate getting equation

$$(60) \quad P'^2(\xi) = -AP^3(\xi) + BP^2(\xi) + c_3P(\xi) + c_4,$$

where

$$(61) \quad A = \frac{2\alpha}{3\beta}, \quad B = \frac{V - 3\beta\kappa^2}{\beta}, \quad c_3 = \frac{2c_1}{\beta}, \quad c_4 = \frac{2c_2}{\beta},$$

and c_2 is a constant of integration. It is well known [10] that Eq. (60) has the Jacobi-elliptic solution

$$(62) \quad P(\xi) = y_1 + (y_2 - y_1) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{A(y_1 - y_2)} \xi; m \right),$$

where $m = \sqrt{\frac{y_1 - y_2}{y_1 - y_3}}$ is the modulus of Jacobi-elliptic function such that y_j , $j = 1, 2, 3$, $y_1 > y_2 > y_3$ are roots of equation $Ay^3 - By^2 - c_3y - c_4 = 0$.

Now, Eq. (1) has the Jacobi-elliptic solution

$$(63) \quad q(x, t) = \left\{ y_1 + (y_2 - y_1) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{A(y_1 - y_2)} \xi; m \right) \right\} e^{i[-\kappa x - 2\beta \kappa^3 t + \sigma W(t)]}.$$

If $m \rightarrow 1^-$, $y_2 \rightarrow y_3$, $\operatorname{sn} \rightarrow \tanh$, then we have the dark soliton solution of Eq. (1) as

$$(64) \quad q(x, t) = \left\{ y_1 - (y_1 - y_3) \tanh^2 \left(\frac{1}{2} \sqrt{A(y_1 - y_3)} \xi \right) \right\} e^{i[-\kappa x - 2\beta \kappa^3 t + \sigma W(t)]},$$

provided $A(y_1 - y_3) > 0$.

5.2. $n = 1$. Let us now solve Eq. (58) at $n = 1$ taking into account the conditions (59). Now, we get the following equation

$$(65) \quad P'^2(\xi) = -\Omega P^4(\xi) + BP^2(\xi) + c_3P(\xi) + c_4,$$

where

$$(66) \quad \Omega = \frac{\alpha}{2\beta}, \quad B = \frac{V - 3\beta\kappa^2}{\beta}, \quad c_3 = \frac{2c_1}{\beta}, \quad c_4 = \frac{2c_2}{\beta}.$$

It is well known [10] that Eq. (65) has a solution in terms of the Jacobian elliptic functions, namely

$$(67) \quad P(\xi) = \frac{y_1(y_3 - y_2) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_3)} \xi; m \right) + y_2(y_1 - y_3)}{(y_3 - y_2) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_3)} \xi; m \right) + y_1 - y_3},$$

where $m = \sqrt{\frac{(y_1 - y_4)(y_3 - y_2)}{(y_4 - y_2)(y_1 - y_3)}}$ is the modulus of the Jacobian elliptic function in which y_j , $j = 1, 2, 3, 4$, $y_1 > y_2 > y_3 > y_4$, are roots of equation $\Omega y^4 - By^2 - c_3y - c_4 = 0$, provided $y_1 + y_2 + y_3 + y_4 = 0$.

Now, Eq. (1) has the Jacobian elliptic solution

$$(68) \quad q(x, t) = \left\{ \frac{y_1(y_3 - y_2) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_3)} \xi; m \right) + y_2(y_1 - y_3)}{(y_3 - y_2) \operatorname{sn}^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_3)} \xi; m \right) + y_1 - y_3} \right\} \times e^{i[-\kappa x - 2\beta \kappa^3 t + \sigma W(t)]},$$

where $\Omega(y_4 - y_2)(y_1 - y_3) > 0$. If $m \rightarrow 1^-$, $y_3 \rightarrow y_4$, $\operatorname{sn} \rightarrow \tanh$, then we have the soliton solution of Eq. (1) as

$$(69) \quad q(x, t) = \left\{ \frac{y_1(y_4 - y_2) \tanh^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_4)} \xi \right) + y_2(y_1 - y_4)}{(y_4 - y_2) \tanh^2 \left(\frac{1}{2} \sqrt{\Omega(y_4 - y_2)(y_1 - y_4)} \xi \right) + y_1 - y_4} \right\} \times e^{i[-\kappa x - 2\beta \kappa^3 t + \sigma W(t)]},$$

provided $\Omega(y_4 - y_2)(y_1 - y_4) > 0$.

6. Conclusions. The current work addressed the RKL equation with power-law of nonlinearity in presence of multiplicative white noise by the usage of Itô calculus. Two integration schemes revealed the soliton solutions. The effect of white noise always appeared in the phase component of the model. The amplitude component remained unaffected with the white noise factor. The results can be later extended further along to additional devices such as in birefringent fibres, DWDM topology, couplers, magneto-optic waveguides, and eventually into meta-optics. Such results are in the works and they will be sequentially disseminated.

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REFERENCES

- [1] ABDELRAHMAN M. A. E., W. W. MOHAMMED, M. ALESEMI, S. ALBOSAILY (2021) The effect of multiplicative noise on the exact solutions of nonlinear Schrödinger equation, *AIMS Mathematics*, **6**, 2970–2980.
- [2] ALBOSAILY S., W. W. MOHAMMED, M. A. AIYASHI, M. A. E. ABDELRAHMAN (2020) Exact solutions of the $(2 + 1)$ -dimensional stochastic chiral nonlinear Schrödinger equation, *Symmetry*, **12**, 1874–1886.
- [3] ARSHED S., A. BISWAS, P. GUGGILLA, A. S. ALSHOMRANI (2020) Optical solitons for Radhakrishnan–Kundu–Lakshmanan equation with full nonlinearity, *Phys. Lett. A*, **384**, 126191.
- [4] BISWAS A. (2009) 1-soliton solution of the generalized Radhakrishnan–Kundu–Lakshmanan equation, *Phys. Lett. A*, **373**, 2546–2548.
- [5] BISWAS A. (2018) Optical soliton perturbation with Radhakrishnan–Kundu–Lakshmanan equation by traveling wave hypothesis, *Optik*, **171**, 217–220.
- [6] BISWAS A., M. EKICI, A. SONMEZOGLU, A. S. ALSHOMRANI (2018) Optical solitons with Radhakrishnan–Kundu–Lakshmanan equation by extended trial function scheme, *Optik*, **160**, 415–427.
- [7] BISWAS A., Y. YILDIRIM, E. YASAR, M. F. MAHMOOD, A. S. ALSHOMRANI et al. (2018) Optical soliton perturbation for Radhakrishnan–Kundu–Lakshmanan equation with a couple of integration schemes, *Optik*, **163**, 126–136.
- [8] GANJI D. D., A. ASGARI, Z. Z. GANJI (2008) Exp-function based solution of nonlinear Radhakrishnan, Kundu and Laskshmanan (RKL) equation, *Acta Appl. Math.*, **104**, 201–209.
- [9] GONZÁLEZ-GAXIOLA O., A. BISWAS (2019) Optical solitons with Radhakrishnan–Kundu–Lakshmanan equation by Laplace–Adomian decomposition method, *Optik*, **179**, 434–442.

- [¹⁰] KUDRYASHOV A. N. (2021) The Radhakrishnan–Kundu–Lakshmanan equation with arbitrary refractive index and its exact solutions, *Optik*, **238**, 166738.
- [¹¹] MOHAMMED W. W., H. AHMAD, A. E. HAMZA, E. S. ALY, M. EL-MORSHEDY et al. (2021) The exact solutions of the stochastic Ginzburg–Landau equation, *Results in Physics*, **23**, 103988.
- [¹²] MOHAMMED W. W., H. AHMAD, H. BOULARES, F. KHELI, M. EL-MORSHEDY (2021) Exact solutions of Hirota–Maccari system forced by multiplicative noise in the Itô sense, *Journal of Low Frequency Noise, Vibration and Active Control*, **41**(1), 74–84, DOI: 10.1177/14613484211028100.
- [¹³] MOHAMMED W. W., N. IQBAL, A. ALI, M. EL-MORSHEDY (2021) Exact solutions of the stochastic new coupled Konno–Oono equation, *Results Phys.*, **21**, 103830.
- [¹⁴] MOHAMMED W. W., M. EL-MORSHEDY (2021) The influence of multiplicative noise on the stochastic exact solutions of the Nizhnik–Novikov–Veselov system, *Math. Comput. Simul.*, **190**, 192–202.
- [¹⁵] MOHAMMED W. W., S. ALBOSAILY, N. IQBAL, M. EL-MORSHEDY (2021) The effect of multiplicative noise on the exact solutions of the stochastic Burger equation, *Waves Random Complex Media*, 13 pp, <https://doi.org/10.1080/17455030.2021.1905914>.
- [¹⁶] MOHAMMED W. W., M. ALESEMI, S. ALBOSAILY, N. IQBAL, M. EL-MORSHEDY (2021) The exact solutions of stochastic fractional-space Kuramoto–Sivashinsky equation by using (G'/G) -expansion method, *Mathematics*, **9**, 2712.
- [¹⁷] YILDIRIM Y., A. BISWAS, M. EKICI, H. TRIKI, O. GONZÁLEZ-GAXIOLA et al. (2020) Optical solitons in birefringent fibers for Radhakrishnan–Kundu–Lakshmanan equation with five prolific integration norms, *Optik*, **208**, 164550.
- [¹⁸] ZAYED E. M. E., R. M. A. SHOHIB (2019) Optical solitons and other solutions to Biswas–Arshed equation using the extended simplest equation method, *Optik*, **185**, 626–635.
- [¹⁹] ZAYED E. M. E., K. A. GEPREEL, M. E. M. ALNGAR (2021) Addendum to Kudryashov’s method for finding solitons in magneto-optics waveguides to cubic-quartic NLSE with Kudryashov’s sextic power law of refractive index, *Optik*, **230**, 166311.
- [²⁰] ZAYED E. M. E., R. M. A. SHOHIB, M. E. M. ALNGAR, A. BISWAS, Y. YILDIRIM et al. (2022) Optical solitons in the Sasa–Satsuma model with multiplicative noise via Itô calculus, *Ukr. J. Phys. Opt.*, **23**(1), 9–14.

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