

NUMERICAL SIMULATION OF HIGHLY DISPERSIVE
DARK OPTICAL SOLITONS WITH KERR LAW OF
NONLINEAR REFRACTIVE INDEX BY
LAPLACE–ADOMIAN DECOMPOSITION METHOD

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Abstract

This work is a numerical perspective to highly dispersive dark optical solitons by using Laplace-Adomian decomposition method. The results are reported using this scheme with highly precise accuracy and the error measure is stunningly low. The surface plots, density plots and error plots are exhibited for different parameter choices. The simulations are almost an exact replica of such solitons that analytically arise from the governing system. The suggested iterative scheme finds the solution without any discretization, linearization, or restrictive assumptions.

Key words: nonlinear Schrödinger equation, highly dispersive dark solitons, Kerr type of nonlinearity, optical fibre, Laplace-transform, Adomian polynomials

2020 Mathematics Subject Classification: 78A60

1. Introduction. In 2019, the notion of highly dispersive optical solitons was introduced. This concept was conceived out of extreme necessity. The two key essential features for a stable transmission of solitons through optical fibres across transcontinental and transoceanic distances are chromatic dispersion (CD) and self-phase modulation (SPM). If, somehow, this balance gets compromised

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at any point during transmission, due to fibre irregularities or other technical glitches, it would result in preposterous and catastrophic havoc. To prevent such a situation, it is always necessary to assure that the balance is strictly maintained.

Occasionally, there are situations when CD runs low. Therefore, it is imperative to include additional dispersion terms that would compensate for the low dispersion count that stems from CD. To this effect, inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD) are included to account for such a low CD. This led to the concept of highly dispersive (HD) optical solitons first learned about in 2019 [1–5]. Incidentally, it is not out of place to mention that later, HD solitons with eighth-order dispersion have been proposed and successfully analyzed [6].

There are definitely a few issues in this context. It must be noted that although the study is being carried out with IMD, soliton studies are conducted in single-mode fibres while IMD typically arises in the context of multi-mode fibres. However, this technical issue is disregarded for the mathematical analysis in this work. The inclusion of these six dispersion terms would lead to considerable soliton radiation and that would slow down the solitons. However, this issue is overlooked so that the focus is to handle the HD soliton by the aid of a very efficient decomposition method, which results from the combination of the well-known Adomian decomposition method with the usual Laplace transform and is named as the Laplace-Adomian decomposition method (LADM).

HD solitons have been analytically studied in the past with four different forms of SPM. They are Kerr type, quadratic-cubic type, polynomial type and non-local type of nonlinearity [7]. For each of these cases, analytical studies along with numerical schemes using LADM have been studied and those results have been reported. Their conservation laws for polarization-preserving fibres and birefringent fibres have been recovered and reported [8–11]. The current work will bridge the gap of numerically addressing dark HD solitons with Kerr type of nonlinear refractive index using LADM. Incidentally, bright HD optical solitons with Kerr type of nonlinearity have already been studied [12]. The results of this paper will thus complete the study of HD solitons numerically for polarization-preserving fibres. The detailed analysis is presented in the remaining part of the article after a brief and precise introduction to the model to be studied.

2. The model. The dimensionless nonlinear Schrödinger equation (NLSE) which includes dispersion terms of all orders and has Kerr type of nonlinearity is [13–15]:

$$(1) \quad iq_t + ia_1q_x + a_2q_{xx} + ia_3q_{xxx} + a_4q_{xxxx} + ia_5q_{xxxxx} + a_6q_{xxxxxx} + b|q|^2q = 0,$$

where q is a complex-valued function of the real dependent variables x (spatial) and t (temporal), moreover $i^2 = -1$. Linear temporal evolution is represented by the first term. Dispersion terms which make the solitons highly dispersive

are stood for by the next six terms. These are yielded by the coefficients of a_s with $1 \leq s \leq 6$ that are inter-modal dispersion (IMD), group velocity dispersion (GVD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD), respectively. In addition, the coefficient b indicates the self-phase modulation factor based on cubic or Kerr type of nonlinearity.

2.1. Dark soliton solutions for the model (1). In order to consider the proposed mathematical model (1) we assume, following recent research reported in [13,14] that dark solitons are given by the following function

$$(2) \quad q(x, t) = A \left(\tanh[B(x - \eta t)] + \tanh^3[B(x - \eta t)] \right) e^{i[-\kappa x + \omega t + \theta_0]},$$

where A and B are the amplitude of the soliton and its inverse width, respectively, which will be established below. In addition, κ is the wave number of the soliton, ω is the frequency, while θ_0 is the phase constant.

The dependence between the velocity of the soliton η with the coefficients of the system and its wave number κ is given by

$$(3) \quad \eta = a_1 - 2a_2\kappa - 3a_3\kappa^2 + 4a_4\kappa^3 + 5a_5\kappa^4 - 6a_6\kappa^5.$$

In [15], the amplitude A was established as a function of the inverse soliton width as:

$$(4) \quad A = \sqrt{\frac{182.946a_6B^6 - 16.874P_3B^4 + 11.847P_2B^2}{26.118b}}.$$

The inverse width indicated by B is obtained from the following sixth degree polynomial equation through its real roots

$$(5) \quad 670.8136a_6B^6 - 59.059P_3B^4 + 35.541P_2B^2 - 9933P_1 = 0$$

whose coefficients are reported in [15] as

$$(6) \quad P_1 = \kappa(a_1 - a_2\kappa - a_3\kappa^2 + a_4\kappa^3 + a_5\kappa^4 - a_6\kappa^5) - \omega$$

$$(7) \quad P_2 = a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4$$

$$(8) \quad P_3 = a_4 + 5a_5\kappa - 15a_6\kappa^2.$$

Additionally, the coefficients of the model and the soliton wave number κ must simultaneously satisfy the following pair of algebraic equations [14]

$$\begin{cases} 4a_4\kappa + 10a_5\kappa^2 - 20a_6\kappa^3 - a_3 = 0 \\ 6a_6\kappa - a_5 = 0. \end{cases}$$

Finally, the constraint:

$$(9) \quad (182.946a_6B^6 - 16.874P_3B^4 + 11.847P_2B^2)(26.118b) > 0$$

must be satisfied.

3. Recapitulation of LADM and application to the model. The non-linear Schrödinger equation given by model (3) will now be solved by means of LADM. The method that we will present below has been used successfully to solve nonlinear partial differential equations and its foundations can be found in [16,17].

We will begin our analysis by separating into real and imaginary parts and therefore considering that the soliton-type solution for Eq. (1) are given as $q = u + iv$. Then our model given by Eq. (1) will be decomposed into

$$(10) \quad u_t = -a_1 u_x - a_2 v_{xx} - a_3 u_{xxx} - a_4 v_{xxx} - a_5 u_{xxxx} - a_6 v_{xxxx} - bv(u^2 + v^2)$$

$$(11) \quad v_t = -a_1 v_x + a_2 u_{xx} - a_3 v_{xxx} + a_4 u_{xxx} - a_5 v_{xxxx} + a_6 u_{xxxx} + bu(u^2 + v^2)$$

assuming $\Re(q(x, t))_{t=0} = u(x, 0)$ and $\Im(q(x, t))_{t=0} = v(x, 0)$ as the initial conditions.

In an operator form, Eqs. (10) and (11) become

$$(12) \quad D_t u = -a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u, v)$$

$$(13) \quad D_t v = -a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v),$$

where the operator D_t means the time derivative, whereas that D_x^j is a differential operator with respect to the spatial variable x of j -th order and N_l is a high order differential operator in two variables for $l = 1, 2$, which is nonlinear.

If we apply the well-known Laplace transform, which we will denote from now on as $\mathcal{L}$ on the two sides of (12) and (13) and considering the condition at the initial point we get

$$(14) \quad u(x, s) = \frac{1}{s} \left[u(x, 0) + \mathcal{L} \{ -a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u, v) \} \right]$$

$$(15) \quad v(x, s) = \frac{1}{s} \left[v(x, 0) + \mathcal{L} \{ -a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v) \} \right].$$

The components u and v are easily obtained by applying the inverse Laplace transform $\mathcal{L}^{-1}$ on both sides of Eqs. (14) and (15):

$$(16) \quad u(x, t) = u(x, 0) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ -a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u, v) \} \right]$$

$$(17) \quad v(x, t) = v(x, 0) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ -a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v) \} \right].$$

Then, the solution function $q = u + iv$, which is obtained by LADM is assumed to be given by a series form

$$(18) \quad q = u + iv = \sum_{n=0}^{\infty} u_n + i \sum_{n=0}^{\infty} v_n.$$

In practice, all terms of the series u_n and v_n cannot be determined, so we have an approximation of the solution by the next approximate sums

$$(19) \quad q_M = u_M + iv_M = \sum_{n=0}^{M-1} u_n + i \sum_{n=0}^{M-1} v_n,$$

where

$$(20) \quad \lim_{M \rightarrow \infty} u_M(x, t) = u(x, t), \quad \lim_{M \rightarrow \infty} v_M(x, t) = v(x, t).$$

A very important point of this method is that it works very successfully for non-linear differential equations. Using LADM the nonlinear terms N_1 and N_2 are decomposed as

$$(21) \quad N_1(u, v) = -b(vu^2 + v^3) = -b \sum_{n=0}^{\infty} A_n(u_0, \dots, u_n; v_0, \dots, v_n)$$

and

$$(22) \quad N_2(u, v) = b(uv^2 + u^3) = b \sum_{n=0}^{\infty} B_n(u_0, \dots, u_n; v_0, \dots, v_n),$$

where A_n and B_n represent polynomials, namely, the Adomian polynomials in two variables, which are defined as [16]

$$(23) \quad A_n(u_0, \dots, u_n; v_0, \dots, v_n) = \frac{1}{n!} \frac{d^n}{d\mu^n} \left[N_1 \left(\sum_{i=1}^{\infty} \mu^i u_i; \sum_{i=1}^{\infty} \mu^i v_i \right) \right]_{\mu=0},$$

for every $n = 0, 1, 2, \dots$

$$(24) \quad B_n(u_0, \dots, u_n; v_0, \dots, v_n) = \frac{1}{n!} \frac{d^n}{d\mu^n} \left[N_2 \left(\sum_{i=1}^{\infty} \mu^i u_i; \sum_{i=1}^{\infty} \mu^i v_i \right) \right]_{\mu=0},$$

for every $n = 0, 1, 2, \dots$

Inserting (18), (21) and (22) into (16) and (17), we arrive at the following two expressions

$$\begin{aligned} \sum_{n=0}^{\infty} u_n = u(x, 0) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \left\{ - (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) \cdot \sum_{n=0}^{\infty} u_n \right. \right. \\ \left. \left. - (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) \cdot \sum_{n=0}^{\infty} v_n - b \cdot \sum_{n=0}^{\infty} A_n \right\} \right] \end{aligned}$$

$$(25) \quad \sum_{n=0}^{\infty} v_n = v(x, 0) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \left\{ -(a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) \cdot \sum_{n=0}^{\infty} v_n \right. \right. \\ \left. \left. + (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) \cdot \sum_{n=0}^{\infty} u_n + b \cdot \sum_{n=0}^{\infty} B_n \right\} \right].$$

From the above, equating terms of the same order, the following algorithm is obtained to calculate the partial sums u_n and v_n , namely:

$$(26) \quad \begin{aligned} u_0(x, t) &= \Re e(q), \\ u_1(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) u_0 \right. \\ &\quad \left. + (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) v_0 - b A_0 \} \right], \\ u_2(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) u_1 \right. \\ &\quad \left. + (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) v_1 - b A_1 \} \right], \\ &\vdots \\ u_{n+1}(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) u_n \right. \\ &\quad \left. + (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) v_n - b A_n \} \right]. \end{aligned}$$

$$(27) \quad \begin{aligned} v_0(x, t) &= \Im m(q), \\ v_1(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) v_0 \right. \\ &\quad \left. - (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) u_0 + b B_0 \} \right], \\ v_2(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) v_1 \right. \\ &\quad \left. - (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) u_1 + b B_1 \} \right], \\ &\vdots \\ v_{n+1}(x, t) &= -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ (a_1 D_x^1 + a_3 D_x^3 + a_3 D_x^5) v_n \right. \\ &\quad \left. - (a_2 D_x^2 + a_4 D_x^4 + a_6 D_x^6) u_n + b B_n \} \right]. \end{aligned}$$

Considering that $N_1(u, v) = v(u^2 + v^2)$ and $N_2(u, v) = u(v^2 + u^2)$, making use of the definitions established in (23) and (24) the polynomials for the first degrees

are

$$\begin{aligned}
A_0 &= u_0^2 v_0 + v_0^3, \\
A_1 &= 2u_0 u_1 v_0 + u_0^2 v_1 + 3v_0^2 v_1, \\
A_2 &= 2u_0 u_2 v_0 + u_1^2 v_0 + 2u_0 u_1 v_1 + u_0^2 v_2 + 3v_0^2 v_2 + 3v_0 v_1^2, \\
(28) \quad A_3 &= 2u_0 u_3 v_0 + 2u_1 u_2 v_0 + 2u_0 u_2 v_1 + u_1^2 v_1 + 2u_0 u_1 v_2 + u_0^2 v_3 + 3v_0^2 v_3 \\
&\quad + 6v_0 v_1 v_2 + v_1^3, \\
A_4 &= 2u_0 v_0 u_4 + 2v_0 u_1 u_3 + 2u_0 v_1 u_3 + 2u_1 v_1 u_2 + 2u_0 v_2 u_2 + u_1^2 v_2 + 2u_0 v_1 u_3 \\
&\quad + u_0^2 v_4 + 3v_0^2 v_4 + 6v_0 v_1 v_3 + 3v_0 v_2^2 + 3v_1^2 v_2 + v_0 u_2^2,
\end{aligned}$$

and

$$\begin{aligned}
B_0 &= v_0^2 u_0 + u_0^3, \\
B_1 &= v_0^2 u_1 + 3u_0^2 u_1 + 2v_0 v_1 u_0, \\
B_2 &= v_1^2 u_0 + 2v_0 v_1 u_1 + v_0^2 u_2 + 3u_0^2 u_2 + 3u_0 u_1^2 + 2v_0 v_2 u_0, \\
(29) \quad B_3 &= 2v_0 v_3 u_0 + 2v_1 v_2 u_0 + 2v_0 v_2 u_1 + v_1^2 u_1 + 2v_0 v_1 u_2 + v_0^2 u_3 + 3u_0^2 u_3 \\
&\quad + 6u_0 u_1 u_2 + u_1^3, \\
B_4 &= 2v_0 u_0 v_4 + 2u_0 v_1 v_3 + 2v_0 u_1 v_3 + 2v_1 u_1 v_2 + 2v_0 u_2 v_2 + v_1^2 u_2 + 2v_0 u_1 v_3 \\
&\quad + v_0^2 u_4 + 3u_0^2 u_4 + 6u_0 u_1 u_3 + 3u_0 u_2^2 + 3u_1^2 u_2 + u_0 v_2^2.
\end{aligned}$$

To conclude, considering the terms given by Eq. (2), the necessary components for a required level of approximation for both u and v in Eq. (18) will be identified in a relatively simple way; therefore the complete solution $q(x, t)$ of our model given by Eq. (1) can be set up in a simple way. LADM gives us a very powerful approximation technique for solving problems modelled with complex nonlinearities [18].

4. Numerical simulations. In this section, we perform numerical evaluation of the solution for Eq. (1) specifically in the case of dark solitons using LADM with the initial condition $q(x, 0)$, which we can obtain from Eq. (2) by evaluating at $t = 0$ and is given by

$$(30) \quad q(x, 0) = A \left(\tanh[B(x)] + \tanh^3[B(x)] \right) e^{i[-\kappa x + \theta_0]}.$$

T a b l e 1

Coefficients of the mathematical model given by Eq. (1) for dark solitons

Cases	a_1	a_2	a_3	a_4	a_5	a_6	b	κ	ω	θ_0	η	M	Max Error
1	1.16	0.25	0.30	0.40	1.20	1.10	-2.25	0.05	0.40	2.20	1.12	14	1.0×10^{-8}
2	2.20	0.55	0.45	0.25	1.10	2.15	-2.35	0.10	0.75	1.80	2.07	14	5.0×10^{-8}
3	1.35	1.15	0.75	1.33	0.50	1.80	1.65	0.25	0.95	2.55	0.63	14	2.0×10^{-8}
4	0.85	1.15	1.18	0.95	2.50	3.30	2.05	0.01	0.10	1.67	0.82	14	1.5×10^{-8}

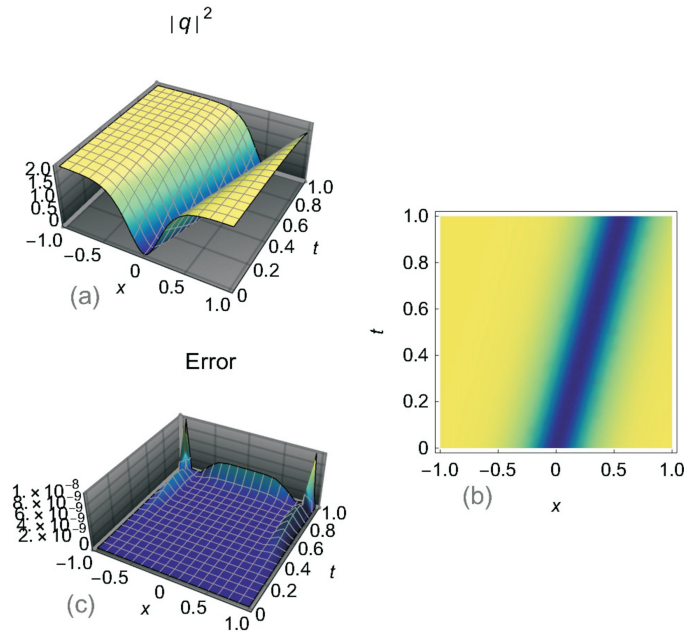


Fig. 1. (a) Dark soliton obtained numerically by LADM, (b) the corresponding density plot, and (c) the absolute error corresponding to the 14-step iteration for Case 1

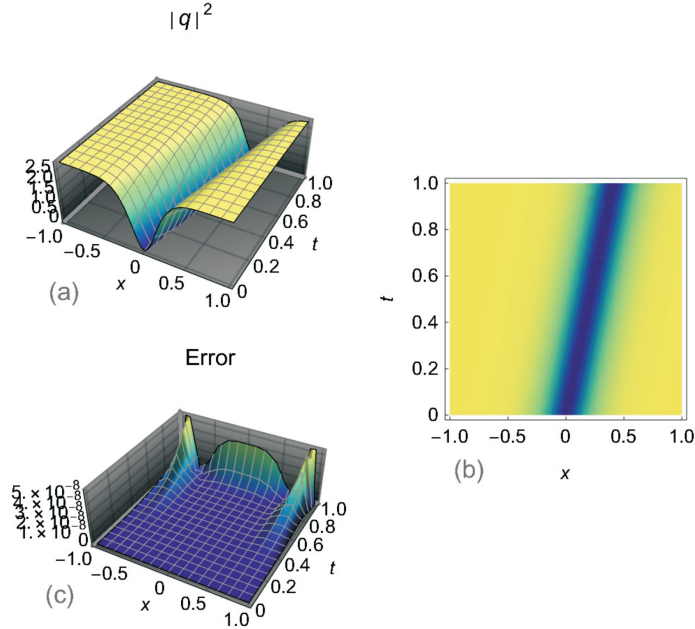


Fig. 2. (a) Dark soliton obtained numerically by LADM, (b) the corresponding density plot, and (c) the absolute error corresponding to the 14-step iteration for Case 2

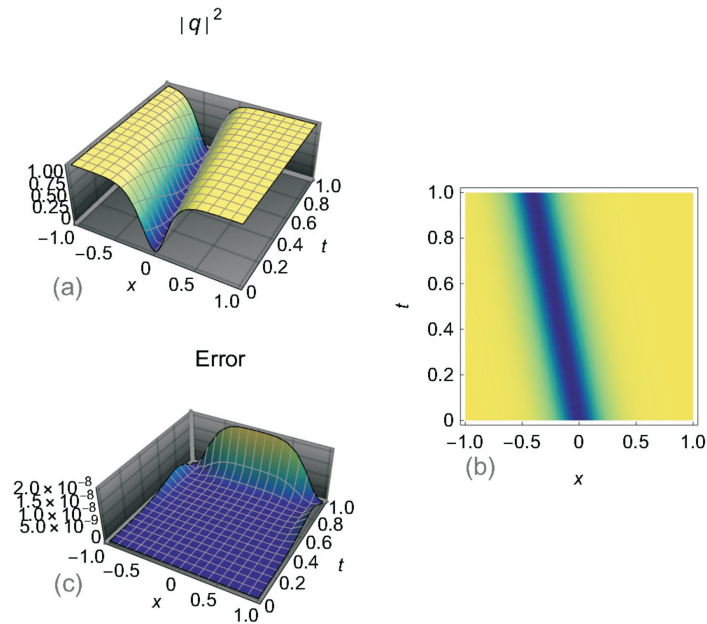


Fig. 3. (a) Dark soliton obtained numerically by LADM, (b) the corresponding density plot, and (c) the absolute error corresponding to the 14-step iteration for Case 3

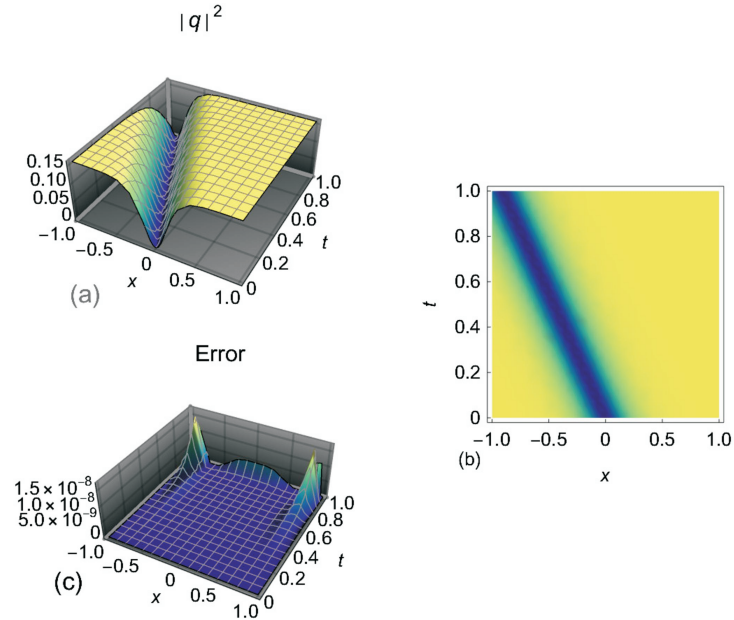


Fig. 4. (a) Dark soliton obtained numerically by LADM, (b) the corresponding density plot, and (c) the absolute error corresponding to the 14-step iteration for Case 4

We will now consider the numerical cases listed in Table 1 in order to perform the corresponding numerical simulations. The results obtained as well as their respective absolute errors resulting from the simulations are shown in Fig. 1, 2, 3, and 4, respectively.

Conclusions. The results of the numerical analysis show HD dark optical soliton propagation through fibres across intercontinental distances. LADM has made this visualization possible. Therefore, it is time to move on with the success of this result. A deluge of opportunities is awaited with HD solitons. One must study HD solitons in birefringent fibres numerically using LADM for all four nonlinear forms. Later, it is also necessary to address HD solitons with absence of SPM, in birefringent fibres. Finally, it is also imperative to extend this entire set of such studies to dispersion flattened fibres. The results of this research work will soon be reported sequentially, and future applications of the same model will be discussed.

REFERENCES

- [¹] BISWAS A., M. EKICI, A. SONMEZOGLU, M. R. BELIC (2019) Highly dispersive optical solitons with Kerr law nonlinearity by exp-function, *Optik*, **185**, 121–125.
- [²] BISWAS A., M. EKICI, A. SONMEZOGLU, M. R. BELIC (2019) Highly dispersive optical solitons in absence of self-phase modulation by exp-function, *Optik*, **186**, 436–442.
- [³] YILDIRIM Y., A. BISWAS, P. GUGGILLA, O. GONZÁLEZ-GAXIOLA, M. EKICI et al. (2020) Exhibit of highly dispersive optical solitons in birefringent fibers with four forms of nonlinear refractive index by exp-function expansion, *Optik*, **208**, 164471.
- [⁴] GONZÁLEZ-GAXIOLA O., A. BISWAS, M. ASMA, A. K. ALZHRANI (2021) Highly dispersive optical solitons with non-local law of refractive index by Laplace-Adomian decomposition, *Opt. Quant. Electron.*, **53**, 55.
- [⁵] GONZÁLEZ-GAXIOLA O., A. BISWAS, M. EKICI, S. KHAN (2022) Highly dispersive optical solitons with quadratic-cubic law of refractive index by the variational iteration method, *J. Opt.*, **51**, 29–36.
- [⁶] KUDRYASHOV N. A. (2020) Highly dispersive optical solitons of the generalized nonlinear eighth-order Schrödinger equation, *Optik*, **206**, 164335.
- [⁷] ZAYED E. M. E., A.-G. AL-NOWEHY, M. E. M. ALNGAR, A. BISWAS, M. ASMA et al. (2021) Highly dispersive optical solitons in birefringent fibers with four nonlinear forms using Kudryashov’s approach, *J. Opt.* **50**, 120–131.
- [⁸] BISWAS A., H. KARA, A. S. ALSHOMRANI, M. EKICI, Q. ZHOU et al. (2019) Conservation laws for highly dispersive optical solitons, *Optik*, **199**, 163283.
- [⁹] BISWAS A., H. KARA, Q. ZHOU, A. K. ALZHRANI, M. R. BELIC (2020) Conservation laws for highly dispersive optical solitons in birefringent fibers, *Regular and Chaotic Dynamics*, **25**, 166–177.
- [¹⁰] BISWAS A., M. EKICI, A. SONMEZOGLU, M. R. BELIC (2019) Highly dispersive optical solitons in absence of self-phase modulation by Jacobi’s elliptic function expansion, *Optik*, **189**, 109–120.

- [¹¹] KUDRYASHOV N. A. (2020) Method for finding highly dispersive optical solitons of nonlinear differential equations, *Optik*, **206**, 163550.
- [¹²] GONZÁLEZ-GAXIOLA O., A. BISWAS, A. S. ALSHOMRANI (2020) Highly dispersive optical solitons having Kerr law of refractive index with Laplace-Adomian decomposition, *Rev. Mex. de Física*, **66**, 291–296.
- [¹³] BISWAS A., M. EKICI, A. SONMEZOGLU, M. R. BELIC (2019) Highly dispersive optical solitons with Kerr law nonlinearity by F -expansion, *Optik*, **181**, 1028–1038.
- [¹⁴] BISWAS A., M. EKICI, A. SONMEZOGLU, M. R. BELIC (2019) Highly dispersive optical solitons with Kerr law nonlinearity by extended Jacobi's elliptic function expansion, *Optik*, **183**, 395–400.
- [¹⁵] KOHL R. W., A. BISWAS, M. EKICI, Q. ZHOU, S. KHAN et al. (2019) Highly dispersive optical soliton perturbation with Kerr law by semi-inverse variational principle, *Optik*, **199**, 163226.
- [¹⁶] ADOMIAN G. (1994) Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers.
- [¹⁷] KHURI S. A. (2001) A Laplace decomposition algorithm applied to class of nonlinear differential equations, *J. Math. Anal. Appl.*, **4**, 141–155.
- [¹⁸] PATEL T., R. MEHER (2017) Thermal analysis of porous fin with uniform magnetic field using Adomian decomposition Sumudu transform method, *Nonlinear Engineering*, **6**, 191–200.

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