NUMERICAL STUDY OF HYDRODYNAMIC
INTERACTION BETWEEN A ROW OF SPHEROIDS
IN A STEADY STREAM OF VISCOUS FLUID

Tsvetan Kotsev

Received on July 2, 2020
Presented by S. Radev, Corresponding Member of BAS, on November 30, 2021

Abstract

This paper presents a numerical simulation of the flow past three equal or different spheroids arranged in line placed in a stream of viscous fluid with uniform velocity parallel to the centre line. The hydrodynamic interaction was studied for Reynolds numbers from 1 to 200, axis ratio 0.1, 0.2, 0.4 and 0.8 and different distances between them. The downstream and middle spheroid face reduced drag compared to a single one and at some values of Re and distances it takes negative values. The assemblage of touching spheroids has lower drag than a single one at some flow parameters.

Key words: viscous flow, numerical simulation, spheroids

1. Introduction. In many engineering applications, chemical processes, energy and spray systems, etc., the hydrodynamic interaction between solid particles or droplets immersed in moving viscous fluid is very important. There are many studies done both by analytical and numerical methods concerning the viscous flow past one or two particles but most of them treat particles with exactly spherical shape [1–8]. The problem of Stokes flow past three spheres was studied by KIM [9], LIANG et al. [10] and KOTSEV [11] numerically for Reynolds numbers up to 200. In aerosol spray systems for example, the droplets are not with spherical shape but with spheroidal one so studies about flow past spheroids, elliptic cylinders, etc., are welcome to understand the real processes more precisely. The

steady flow past two elliptic cylinders and two spheroids for moderate Reynolds numbers was considered numerically by KOTSEV and ZAPRYANOV [12] using the so-called “method of numerical matching of solutions”.

The goal of the present study is to investigate the flow and hydrodynamic interaction between an assemblage of three spheroidal particles arranged in line at Reynolds numbers up to 200.

2. Formulation of the problem and method of solution. Three rigid spheroids are immersed in a uniform stream of viscous fluid. The fluid velocity is V_0 and is parallel to the line connecting the body centres. Reynolds number is defined as $Re = 2a_1.V_0/\nu$, where ν is the kinematic viscosity of the fluid and a_1 is the big or small semiaxis of the upstream oblate or prolate spheroid, respectively. For Reynolds numbers used here, the flow is considered axisymmetric. In the case of oblate spheroids that affect the stream stronger than spheres the results obtained for $Re = 200$ cannot be considered fully symmetric while for prolate spheroids the flow is stable even and for higher Re . The distance between the bodies varies from 0.25 up to 30 times the big semiaxis of the upstream spheroid. The governing equations are continuity and steady Navier–Stokes equation as follows:

$$(1) \quad \nabla \cdot \mathbf{V} = 0$$

$$(2) \quad \rho(\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla p + \mu \nabla^2 \mathbf{V},$$

where $\mathbf{V}$ is the velocity vector, μ is the dynamic viscosity, p is the pressure, ρ is the fluid density.

Boundary conditions are:

– no slip on the spheroid surfaces

$$(3) \quad V_x = 0, \quad V_z = 0,$$

– uniform velocity at the inlet

$$(4) \quad V_x = 0, \quad V_z = 0, \quad V_y = V_0,$$

– zero pressure at the outlet

$$(5) \quad p = 0,$$

– symmetry at the outer boundaries

$$(6) \quad \mathbf{V} \cdot \mathbf{n} = 0.$$

The drag and lift forces are calculated integrating the stress tensor on the body surfaces:

$$(7) \quad F_D = \int_S -p \mathbf{n} \cdot \mathbf{j} dS + \int_s \mathbf{n} \cdot t \cdot \mathbf{j} dS, \quad F_L = \int_S -p \mathbf{n} \cdot \mathbf{k} dS + \int_s \mathbf{n} \cdot t \cdot \mathbf{k} dS,$$

where S denotes the surface of the spheroids, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in y and z direction, respectively, $\mathbf{n}$ is the outward normal vector at the surface and t is the viscous stress tensor.

Non-dimensional drag and lift coefficients C_D and C_L are defined as:

$$(8) \quad C_D = \frac{F_D}{\rho V_0^2 A_{pr}/2}, \quad C_L = \frac{F_L}{\rho V_0^2 A_{pr}/2},$$

where A_{pr} is the area of the spheroid projected on a plane perpendicular to the flow direction.

In our case F_L and C_L are equal to zero because the flow is axisymmetric.

Equations (1)–(2) under boundary conditions (3)–(6) are solved numerically using COMSOL package based on the finite elements method. The outer boundary is set at 20 big diameters far from the line of flow symmetry. Experiments show that this distance is sufficient and moving it further does not give any significant difference in the drag coefficients obtained. In the case of flow past oblate spheroids, the complete mesh consists of 106 110 domain elements and 1308 boundary elements. In order to solve the system of equations Generalized Minimal Residual Method (GMRES) for iteratively solving non-symmetric matrices was used. It has good convergence, solution time and optimal use of the memory.

3. Validation of the computational scheme. In order to test the model few comparisons with available data were done. Oblate spheroids with axis ratio A.R. = 0.999 are considered to be spheres, they were put far from each other and the results are compared with these for a single sphere. The distance between the upstream and middle spheroid is denoted by d_1 , d_2 is the distance between the middle and the downstream spheroid. For large d_1 (when the upstream body could be considered a single one) and $Re = 60$, the structure and the length of the recirculating zones are identical with the experiment [13] – about one sphere diameter. The drag coefficient calculated is in a good agreement with Stokes and Oseen's solutions for small Reynolds numbers as well as with the experimental data for moderate Re . Another comparison was done with the results of MASLIYAH and EPSTEIN [14] for the flow past an oblate spheroid with A.R. = 0.2 and a very good agreement for the drag coefficients is established. Comparison with the flow past two spheres in tandem was done with results in [4, 11] for $Re = 60$ and a distance between particles $d = 2$. Drag coefficients of the upstream and downstream spheroids with A.R. = 0.999 are 1.911 and 0.417 while the ones in [11] at the same flow conditions are 1.891 and 0.454. The difference is about 1% for the upstream body and 8% for the downstream one.

4. Results. 4.1. Flow past three oblate spheroids in line arrangement. For $Re = 1$ the flow between spheroids with A.R. = 0.1 and 0.2 lost its Stokes character and clearly visible zones of back flow arise behind the upstream and middle one. Increasing Re the point of separation moves up and the wake

in the gap grows and enlarges towards the downstream body. For $A.R. = 0.5$ the back flow area behind the second spheroid just begins to form while there is no vortex behind the downstream one for all values of $A.R.$ For spheroids with $A.R. = 0.1, 0.2$ and 0.5 , placed at a distance $d_1 = d_2 = 1$ the vortex behind the downstream body appears for $Re = 7, 10$ and 20 , respectively, while for spheres at the same distances – for $Re = 40$. The total drag of an assemblage of touching spheroids with $A.R. = 0.5$ and for $Re = 60$ is less than the drag of a single spheroid at the same parameters. The drag coefficient of a single spheroid is 1.533 [14], while the total one for touching spheroids calculated here is 0.711 . For fixed distance this effect depends on Re and eccentricity of the bodies. If no eddies exist behind the middle and downstream body, then the drag of the assemblage is higher than the drag of a single body. This effect is valid for up to $d_1 = d_2 = 2$ approx. The variation of the drag coefficients with Reynolds number is shown in Fig. 1.

Fig. 1. Drag coefficients of three oblate spheroids vs. Reynolds number, $A.R. = 0.5$, $d_1 = d_2 = 1$

At small Re oblate spheroids with higher $A.R.$ and placed far from each other show higher total drag than spheroids with lower $A.R.$, i.e. the drag on a sphere is higher than the drag on a circular disk that is in accordance with results in [14]. This fact is explained by higher contribution of the drag due to the friction provoked by the larger skin area when $A.R.$ increases. For higher Re this trend changes to opposite due to the higher intensity of vortices behind the spheroids and increasing the wake bubbles. The value of Re where this tendency changes is between 50 and 60. The middle and downstream spheroids experience a reduced drag in comparison with the upstream one, their drag decreases fast and tends to zero quickly with increasing of Re and sometimes accepts negative values.

In the case of spheroids with small $A.R.$, e.g. 0.1 and 0.2 the middle and downstream ones take negative values for Re about 40 and 80, respectively. The upstream spheroid also changes its drag coefficient – it decreases or increases by approximately 11% according to the drag of a single one and depends strongly on the distance between the bodies. Figure 2 shows the flow picture at $Re=60$ and distances between bodies $d = 0$ and $d = 4$.

Fig. 2. Flow past three spheroids for $Re = 60$, $A.R. = 0.5$: a) $d = 0$; b) $d = 4$

A very small recirculating zone exists in the gaps between the first and second spheroid and there is no such behind the middle body. The reason not to have well-formed eddies in the case of touching bodies could be that the gap is quite narrow and is difficult the fluid from the top layers to mix well with the layers beneath. Figure 3 shows the variation of drag coefficients with the distance between spheroids.

Fig. 3. Variation of the drag coefficients with: a) $d = d_1 = d_2$, b) d_1/d_2

When spheroids are touching, the second and third body have negative drag coefficients. The one of the second body is higher in absolute value than the drag coefficient of the downstream one. Both of them added to the drag of the upstream spheroid make the total drag of the assemblage less than the drag of a single body. Increasing d , the behaviour of C_D changes and at a distance of approximately $d = 5$ the second body begins to show bigger drag than the third one and both of them are now positive. The spheroid in the middle changes the drag coefficient from negative to positive at a distance about 1.5 and the one of the downstream spheroid – at about 0.6. Let us fix the distance between upstream and downstream spheroid and move only the middle one forward and backward towards the other two bodies, respectively. The goal is to see how this reflects on the hydrodynamic interaction between particles and their drag coefficients (Fig. 3b). For $d_1/d_2 = 1$ the second spheroid is just in the middle between the other two. Moving it towards the first one its drag coefficient rapidly decreases while the drag

coefficient of the upstream spheroid increases by approximately 9%. For d_1 about 1.48 ($d_1/d_2 = 0.55$) the drag coefficient of the middle body takes negative values. If the middle spheroid is moving towards the downstream one ($d_1/d_2 > 1$), the drag on the upstream body keeps almost the same value while the drag coefficient of the downstream spheroid decreases but is positive even when the second body touches the third one.

The variation of drag coefficients with Re for fixed $d = d_1 = d_2 = 1$ when the upstream and downstream spheroids are equal but the middle one is twice less is similar to the case of equal spheroids. When $d_1/d_2 = 0$ (upstream and middle spheroid are touching) and $Re = 60$ the upstream spheroid feels almost the same drag ($C_{D1} = 1.307$) as in the case of equal spheroids ($C_{D1} = 1.346$). Increasing the ratio d_1/d_2 , C_{D1} falls very slow and keeps almost a constant value. The same is the tendency of the downstream spheroid whose drag coefficient also decreases with d_1/d_2 and changes very slowly. The behaviour of the drag of the spheroid in the middle is quite different – for $d = 1$ C_{D2} accepts negative values for Re about 28 that is higher than the previous case. When the middle body changes its position its drag changes from negative to positive for d_1/d_2 approximately 0.7 that is higher compared with the case of equal bodies where C_{D2} is negative up to $d_1/d_2 = 0.55$. In other words, the smaller the middle spheroid, the longer the distance from the upstream one at which $C_{D2} < 0$.

4.2. Flow past three prolate spheroids in line arrangement. Prolate spheroids have better streamlined shape and disturb the stream less. As a result, the recirculating zones behind the spheroids appear at higher Reynolds numbers compared with the case of oblate spheroids. For Re close to 80 there are no vortices, a very small one appears for higher Re but only behind the first body. Even for $Re=180$ zones of back flow behind the middle and downstream spheroids are negligibly small. The attractive effect due to the negative drag of the middle and downstream body is weakly expressed. In the case when first and third spheroids are fixed and the middle one is moving the tendency of changing C_D vs. Re, distance d and ratio d_1/d_2 is the same as for oblate spheroids but quite weakly expressed.

5. Conclusions. Hydrodynamic interaction of three equal or different spheroids was studied numerically for Reynolds numbers from 1 to 200. The distance between them is the factor that most strongly influences their hydrodynamic interaction. In some cases, the drag force of the middle and downstream body takes negative values which indicates the presence of an attractive effect between them. At the same time upstream spheroid changes its drag up to 11% compared with a single body. At low Reynolds numbers oblate spheroids with higher A.R. show higher total drag coefficient than spheroids with lower A.R. When the spheroids have different size but their A.R. is the same the calculations show that the smaller the middle spheroid, the larger the distance from the upstream one at which its drag takes negative values. In practice a flow past any

number of spheroids, both oblate and prolate could be simulated.

REFERENCES

- [¹] STOKES G. G. (1851) On the effect of the inertial friction of fluids on the motion of pendulums, Trans. Camb. Phil. Soc., **9**, 8–106.
- [²] STIMSON M., G. B. JEFFEREY (1926) The motion of two spheres in a viscous fluid, Proc. R. Soc. Lond. A, **111**, 110–116.
- [³] TSUJI Y., Y. MORIKAWA, K. TERASHIMA (1982) Fluid Dynamics Interaction between Two Spheres, Int. J. Multiphase Flow, **8**(1), 71–82.
- [⁴] ZAPRYANOV Z., E. TOSHEV (1986) Hydrodynamics and Heat Transfer around Two Separated Spherical Particles, Proc. of the International Heat Transfer Conference 8, ASME, New York, **5**, 2549–2553.
- [⁵] KIM I., S. ELGHOBASHI, W. SIRIGNANO (1993) Three dimensional flow over two spheres placed side by side, J. Fluid Mech., **246**, 465–488.
- [⁶] ZHU C., S. C. LIANG, L. S. FAN (1994) Particle wake effects on the drag force of an interacting particle, Int. J. Multiphase Flow, **20**(1), 117–129.
- [⁷] CHEN R. C., J. L. WU (2000) The flow characteristics between two interactive spheres, J. of Chem. Eng. Science, **55**, 1143–1158.
- [⁸] SCHOUVEILER L., A. BRYDON, T. LEWEKE, M. C. THOMPSON (2004) Interactions of the wakes of two spheres placed side by side, European Journal of Mechanics B/Fluids, **23**, 137–145.
- [⁹] KIM S. (1987) Stokes flow past three spheres: an analytical solution, Phys. Fluids, **30**, 2309–2314.
- [¹⁰] LIANG S. C., T. HONG, L. S. FAN (1996) Effects of particle arrangements on the drag force of a particle in the intermediate flow regime, Int. J. Multiphase Flow, **22**(2), 285–306.
- [¹¹] KOTSEV T. (2018) Viscous flow around spherical particles in different arrangements, MATEC Web of Conferences, **145**, 03008.
- [¹²] KOTSEV T., Z. ZAPRYANOV (1987) Hydrodynamic interaction between two oblate spheroids in a uniform viscous flow at moderate Reynolds numbers, Theoretical and applied mechanics, Sofia, XVIII (3), 36–43.
- [¹³] VAN-DYKE M. (1982) An Album of Fluid Motion, The parabolic Press, Stanford California.
- [¹⁴] MASLIYAH J. H., N. EPSTEIN (1970) Numerical study of steady flow past spheroids, J. Fluid Mech., **44**, part 3, 493–512.

Department of Mathematics and Informatics
Sofia University “St. Kliment Ohridski”
5, James Bourchier Blvd
1164 Sofia, Bulgaria
e-mail: tkotsev@fmi.uni-sofia.bg