

NEW GEOMETRY MODELS FOR CALCULATING
TORTUOSITY OF FLOW PATHS IN 3D POROUS MEDIA

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Abstract

The pore tortuosity of flow paths in porous media plays a crucial role in the simulation of various petrophysical properties (e.g., absolute permeability). A geometric model composed of tortuous stream tubes in the three-dimensional porous media with cubic particles (grains) representing solid grains to calculate tortuosity is proposed under the assumption that the particles are allowed to overlap unrestrictedly and fluids are incompressible. The model is formulated as a function of porosity and contains no empirical constants, which helps to reveal the physical mechanism of tortuous pore paths for fluids to flow within porous media. The result performs better in approximating the results on three-dimensional porous rocks.

Key words: tortuosity, fractal dimension, porosity, porous media

Introduction. The prediction of the ability for any fluid to flow through porous media, i.e., (absolute) permeability, is of great importance to many research fields such as petroleum engineering [1], nuclear waste disposal [2], and geothermal exploration [3]. The transmission of porous media is usually closely related to the tortuosity of the tortuous flow path and this geometric parameter, however, is inherently related to the complex microstructure of porous media, resulting in the extreme difficulty in its determination in practice.

The tortuosity of a pore path/channel in a porous medium is defined as [4]:

$$(1) \quad \tau = \frac{L_t}{L_0},$$

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where L_t and L_0 are the actual length of a flow path and the straight length between the two terminals of the flow path.

In recent years, the study of the tortuosity of tortuous flow paths in porous media has attracted extensive attention as it is widely recognized as a flow dominant factor. Some researchers focus on measuring tortuosity by experiments [5], others employ numerical methods to obtain analytical solutions [6]. Representatively, COMITI and RENAUD [7] measured tortuosity through experiments of fluid flow through beds packed with spherical particles, and found that the tortuosity is logarithmically correlated to the reciprocal of porosity with an empirical constant, i.e.,

$$(2) \quad \tau = 1 + C \ln \left(\frac{1}{\phi} \right),$$

where τ is tortuosity, ϕ is the porosity, C is an empirical constant. Regarding the idealized porous media composed of square and spherical particles only, the corresponding C is 0.63 or 0.41, respectively [7].

To better understand the tortuosity mechanism of flow paths in porous media, mathematical models of tortuosity are highly in demand. As a part of the relevant effort, YU and LI [8] present a type of tortuosity mechanism model through analyzing the flow paths in a porous medium composed of idealized square particles. The model is expressed as a function of porosity and there is no empirical constant required:

$$(3) \quad \tau = \frac{1}{2} \left[1 + \frac{1}{2\varphi} + \frac{\sqrt{(\varphi - 1)^2 + 1/4}}{\varphi - 1} \right],$$

where $\varphi \equiv 1/\sqrt{1-\phi}$, τ is the tortuosity, ϕ is the porosity. YUN et al. [9] extended the model from two-dimensional into three-dimensional, and obtained a more comprehensive tortuous geometry model.

KOU et al. [10] analyzed the tortuosity of streamlines in porous media by coupling circular and square particles into models and obtained the relationship between tortuosity and porosity by leveraging statistical methods. The tortuosity models aforementioned are profoundly based on averaging the tortuosities at several representative special positions of flow paths, and consequently are not universal and cannot be applied directly to natural porous media. Therefore, these models are still far from suitable for realistic scenarios in natural materials. Based on Yu and Li's model [8], ZHAO [11] derived the average tortuosity (Eq. (4)) from the integral median value theorem and the calculation accuracy of the tortuosity model is improved considerably. But the formula is only accurate in the two-dimensional porous media model.

$$(4) \quad \bar{\tau} = \frac{1}{2} + \frac{1}{4\varphi} + \frac{2\gamma + 1}{8\sqrt{\gamma^2 - 1/4}} + \frac{\sqrt{\gamma^2 - 1/4}}{2} \cdot \ln \frac{1/2 + \gamma}{\sqrt{\gamma^2 - 1/4}},$$

where $\gamma \equiv \sqrt{1/4 + (\varphi - 1)^2}$, $\varphi \equiv 1/\sqrt{1 - \phi}$, ϕ is the porosity.

Therefore, in this paper, a 3D average tortuosity model of porous media is established by combining the geometric method with the integral median value theorem. And then, the results predicted by the presented model are systematically compared with existing experimental measurements and analytical modelling outcomes.

New tortuosity model for 3D porous media. This work focuses on the porous media that are modelled by cubic particles (grains) separating pores. In principle, the tortuosity can be characterized by averaging the measured values for all possible configurations of flow paths in a porous medium. Similar to Yu and Li [8], this work considers two idealized configurations, i.e., solid particles allowed to either contact or overlap. The two possible configurations of the Newtonian incompressible fluid flowing through porous media can be idealized as the cases shown in Fig. 1 and 2.

The red square in Fig. 1 and 2 is regarded as universal unit cell containing a cubic. $A, B, \dots$ stand for different positions, and the arrows represent flow paths. Figure 2(a) shows ideal flow paths with particles allowed to overlap randomly (the overlapping is not illustrated here just for a clear visualization), where a is side length of the cubic particle and d is the width of pores between grains (i.e., pore size). Since overlapping may occur at any position within a unit cell, a streamline might have different pathways. For instance, $B \rightarrow C \rightarrow D$ is one of the alternative paths. The position C locates on a lateral wall that is perpendicular to the horizontal direction and can be altered arbitrarily (i.e., in Fig. 2(b)), C can appear anywhere on the gray side of the cubic particle. The lateral view of the grey area is shown in Fig. 2(c) at will, and position D is on an edge of the wall where position C is located and is on the same line as position C and position

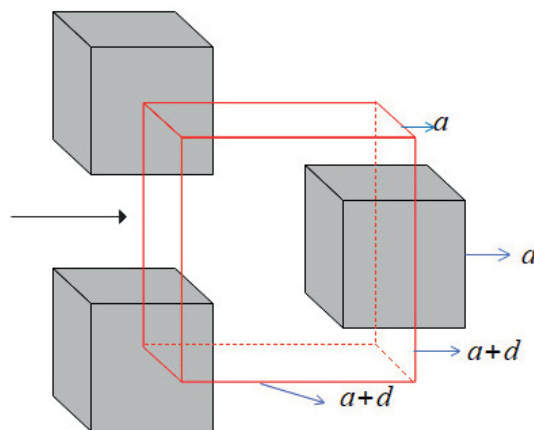


Fig. 1. Three-dimensional configuration for cubic particles and the corresponding unit cell

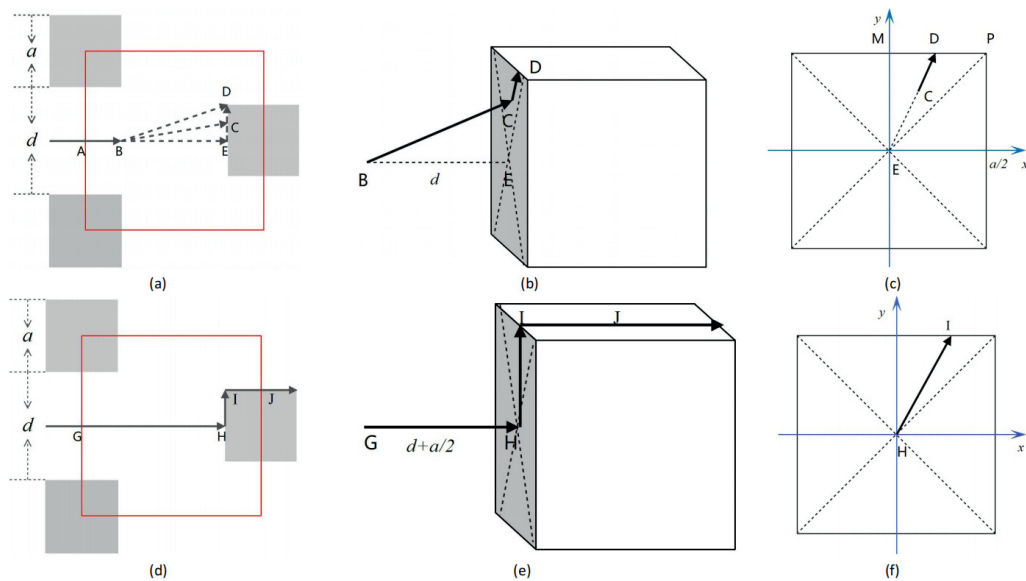


Fig. 2. Lateral view of streamlines flowing around cubic particles and two idealized streamlines for creeping flow in porous media

E. Figure 2(d) illustrates another flow path along which there is no overlapping between particles. $G \rightarrow H \rightarrow I \rightarrow J$ in Fig. 2(d) is a representative flow path among the unit cells. Figure 2(e) and (f) show some dominant streamlines at different angles. H locates right in the centre of the side wall of a particle, and I is an arbitrary endpoint on an edge of the arrow starting from H .

The length, height and thickness of the unit cell as shown in Fig. 1 are $a + d$, $a + d$ and a , respectively. So, the porosity in this unit cell is:

$$(5) \quad \phi = \frac{(a + d)^2 a - a^3}{(a + d)^2 a} = 1 - \left(\frac{1}{1 + d/a} \right)^2.$$

From Eq. (5), we obtain:

$$(6) \quad \frac{d}{a} = \frac{1}{\sqrt{1 - \phi}} - 1.$$

It is clear from Eq. (6) that as $\phi \rightarrow 1$, we have $d \rightarrow \infty$, or oppositely when $\phi \rightarrow 0$, the pore size tends to zero. Namely, when $\phi \rightarrow 1$, there is no space for particles to occupy, which indicates that the pore size turns to be infinite.

Considering the incompressible Newtonian fluid and the thin fluid layer on particle surfaces, the pore tortuosity of a porous medium can then be measured by averaging the tortuosities of all flow paths, i.e.:

$$(7) \quad \bar{\tau} = \frac{1}{n} \sum_i \tau_i.$$

Here n is the total number of all possible fluid flow paths in consideration, and τ_i is tortuosity of the i -th flow path.

The particles in natural porous media can be regarded as randomly distributed and might overlap each other – some particles overlap meanwhile others do not. Yu and Li [8] considered “unrestricted overlapping of particles” and studied streamlines’ tortuosity. They drew a conclusion that the period of streamlines may be in the scale of pore size BE , $B \rightarrow C \rightarrow D$ is one of the fluid paths, and the radial length of the flow is $l_{BE} = d$ as shown in Fig. 2(a). For the first configuration (i.e., Fig. 2(a)–(c)), according to the symmetry of the cubic particle, we consider that location C lies within $\triangle EMP$. Let us assume that the coordinates of point C are (x, y) , where $0 \leq x \leq a/2$, $0 \leq y \leq a/2$.

(i) When $y > 0$, C will not coincide with the centre position E (coordinate origin). Therefore, the length $h(x, y)$ of the streamline $B \rightarrow C \rightarrow D$ between B and E can be expressed as:

$$(8) \quad h(x, y) = l_{BC} + l_{CD} = \sqrt{d^2 + x^2 + y^2} + \frac{a\sqrt{x^2 + y^2}}{2y} - \sqrt{x^2 + y^2}.$$

According to the integral mean value theorem [12], the expectation of $h(x, y)$ within $\triangle EMP$ can be computed by:

$$(9) \quad h(\hat{x}, \hat{y}) = \frac{1}{S_{\triangle EMP}} \iint_{\triangle EMP} h(x, y) \, dx \, dy = a\sqrt{\Phi^2 + 1/8} + \frac{5a}{12} \left(\sqrt{2} + \ln(1 + \sqrt{2}) \right),$$

where $S_{\triangle EMP}$ stands for the area of $\triangle EMP$, $\Phi = d/a$, $h(\hat{x}, \hat{y})$ is determined by porosity ϕ only, which represents the average length of all flow paths between B and D passing through variable positions at C along the unit cell wall. According to the definition Eq. (1), we obtain the tortuosity as:

$$(10) \quad \tau_a = \frac{h(\hat{x}, \hat{y})}{d} = \frac{\sqrt{\Phi^2 + 1/8}}{\Phi} + \frac{5}{12\Phi} \left(\sqrt{2} + \ln(1 + \sqrt{2}) \right).$$

(ii) When $y = 0$, then x can only be 0 within $\triangle EMP$, that is, C and E (coordinate origin) coincide. In this case, the configuration of Fig. 2(b) and (c) becomes the situation shown in Fig. 3. The coordinates of point D can be assumed to be $(x, a/2)$, $0 \leq x \leq a/2$.

$$(11) \quad l_{CD} = g(x) = \sqrt{x^2 + a^2/4}.$$

According to the integral mean value theorem [12], the expectation of $g(x)$ within $\triangle EMP$ can be computed by:

$$(12) \quad g(x_0) = \frac{2}{a} \int_0^{a/2} g(x) \, dx = \frac{\sqrt{2}}{4}a + \frac{a}{4} \ln(1 + \sqrt{2}),$$

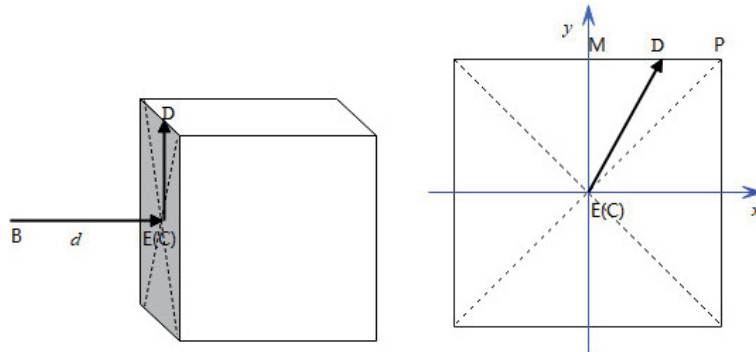


Fig. 3. The configuration when position C coincides with position E

where $g(x_0)$ represents the average length of all flow paths between C and D .

Therefore, the average length $l_{B \rightarrow C \rightarrow D}$ of streamlines between locations B and D through C can be expressed as:

$$(13) \quad l_{B \rightarrow C \rightarrow D} = d + \frac{\sqrt{2}}{4}a + \frac{a}{4}\ln(1 + \sqrt{2}).$$

According to the definition Eq. (1), we obtain the tortuosity as:

$$(14) \quad \tau_b = \frac{l_{B \rightarrow C \rightarrow D}}{d} = 1 + \frac{\sqrt{2} + \ln(1 + \sqrt{2})}{4\Phi},$$

where $\Phi = \frac{d}{a} = \frac{1}{\sqrt{1-\phi}} - 1$.

Combining the above two situations and according to Eq. (7), we have the average tortuosity from Fig. 2(a)–(c) as:

$$(15) \quad \bar{\tau}_1 = \frac{\tau_a + \tau_b}{2} = \frac{1}{2} + \frac{\sqrt{2} + \ln(1 + \sqrt{2})}{3\Phi} + \frac{\sqrt{\Phi^2 + 1/8}}{2\Phi}.$$

Regarding the second configuration (i.e., Fig. 2(d)–(f)), the streamline that has no surrounding particles can be assumed to be an ideal flow path.

Similar to the derivation of Eq. (12), the average length $\widehat{l}_{HI}$ of streamline $H \rightarrow I$ (see Fig. 2(d)–(f)) is equal to $g(x_0)$. The whole length of the streamline $G \rightarrow H \rightarrow I \rightarrow J$ should be taken into account in average tortuosity, therefore, the average tortuosity $\bar{\tau}_2$ in the case is

$$(16) \quad \bar{\tau}_2 = \frac{l_{GH} + l_{IJ}}{l_{GH} + \widehat{l}_{HI} + l_{IJ}} = 1 + \frac{\sqrt{2} + \ln(1 + \sqrt{2})}{4(1 + \Phi)}.$$

According to Eq. (1), the average tortuosity of the flow paths/streamlines $\bar{\tau}$ in the three-dimensional porous media with different configurations of cubic particles

can be obtained by averaging the average tortuosity in the above two cases.

$$(17) \quad \bar{\tau} = \frac{\bar{\tau}_1 + \bar{\tau}_2}{2} = \frac{3}{4} + \frac{\sqrt{\Phi^2 + 1/8}}{4\Phi} + \left(\frac{1}{8(1-\Phi)} + \frac{1}{6\Phi} \right) \left(\sqrt{2} + \ln(1 + \sqrt{2}) \right).$$

Eq. (17) describes the tortuosity as a function of porosity only, and hence there are no extra empirical constants needed to be measured in advance. The equation shows that when $\phi \rightarrow 1$, $\bar{\tau} \rightarrow 1$ and when $\phi \rightarrow 0$, $\bar{\tau} \rightarrow \infty$ which is consistent with the physical situation.

Model comparison. In Figure 4, the predicted results by the tortuosity model (Eq. (17)) presented in this work are compared with those obtained by other empirical or analytical models (Eqs (2)–(4)). Regarding Eq. (2), the parameter C is set as 0.41 for circular particles, 0.63 for square particles [7], and 1.6 for parallelepipedal particles [9]. Eq. (3) was derived for porous media under the two ideal configurations discussed in [8], and Eq. (4) was derived by Zhao [11].

From Figure 4, we can see that the tortuosity values calculated by our new model are in good agreement with many previous models especially when porosity is higher than 0.4. The results of the newly derived three-dimensional cubic

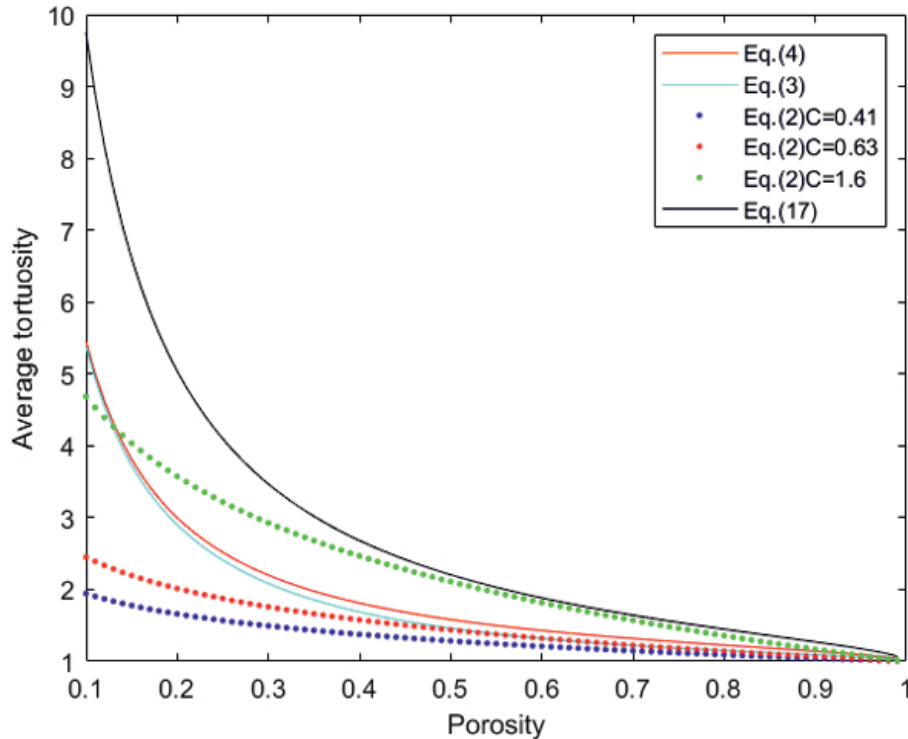


Fig. 4. A comparison between the present model and others in terms of the tortuosity against porosity

particle tortuosity model (Eq. (17)) and the three-dimensional parallelepiped particle model (Eq. (2), $C = 1.6$) are significantly higher than those of other models, and when the porosity is greater than 0.4, the results of the two models are relatively close. This is because in the derivation of Eq. (17), the length of all ideal streamline lines is averaged through the integral mean value theorem, and the resulting tortuosity is closer to the tortuosity of actual porous media. It reflects more complicated streamline tortuosity, so the result of Eq. (17) is the highest.

Conclusions. In this work, we propose a pore tortuosity model for porous media. Different structural configurations (Fig. 1) are obtained under the assumption that particles are allowed to overlap unrestrictedly, and based on the integral median value theorem, an analytical model (Eq. (17)) of bending flow in porous media of square particles is successfully developed. The average tortuosity in a porous medium can be expressed as a function of porosity and there are no other empirical constants needed. The prediction results of the tortuosity model (Eq. (17)) are in a good agreement with the other models results, and can be considered superior to previous geometric models (i.e., Eqs (2)–(4)).

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