

ASSESSMENT OF UNDRAINED SOIL STRENGTH USING THE FALL CONE TEST AND EMPIRICAL MODELLING

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Abstract

Typical empirical correlations in geotechnical practice can be useful for estimating the values of necessary parameters to be used in design and geotechnical calculations. For fine-grained soils, undrained shear strength is a key parameter recommended by Eurocode 7 and is strongly influenced by water content. This paper investigates the relationship between water content, fall cone penetration, and undrained shear strength of medium-plasticity silty clays based on an extensive laboratory dataset obtained over a wide range of moisture conditions, including values both below and above natural water content. The analysis is conducted on remoulded soil samples. Power-law relationships between water content and cone penetration were established, and the corresponding parameters were evaluated and compared with values reported in the literature. The undrained shear strength-water content relationship was further analyzed using normalized expressions based on the water content ratio, enabling direct comparison with existing empirical models. A within-sample regression approach was applied to account for repeated measurements on individual soil samples, revealing consistently high coefficients of determination and a narrow range of regression slopes across materials. The results demonstrate that systematic laboratory testing combined with appropriate data normalization and within-sample analysis provides physically meaningful and robust empirical relationships. A site-specific empirical model for estimating undrained

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shear strength is proposed, offering improved reliability compared to generalized correlations and supporting more accurate site-specific geotechnical assessments.

Key words: penetration, fall cone, undrained shear strength, water content, correlations

Introduction. Water plays one of the key roles in describing the fundamental geotechnical properties and characteristics especially when discussing silty clay soils. Variations in water content in the soil influence changes in its consistency state and significantly alter its behaviour, particularly its shear strength. Two main consistency limits are used to interpret soil behaviour: the plastic limit (w_P) and the liquid limit (w_L). The plastic limit represents the minimum water content at which soil deforms plastically without breaking, while the liquid limit represents the minimum water content at which soil behaves like a liquid, where the shear strength is very low, typically ranging between 0.8 and 2.8 kPa [1–3]. These values are used for fine-grained soils to indirectly define the properties of clay fractions. Additionally, they are used to calculate the plasticity index, which can be empirically related to many soil properties. Many researchers interpret the liquid and plastic limits as representing characteristic soil behaviour under specific stress conditions.

Clayey soils in nature are generally in a solid or plastic state and possess a certain undrained strength. When the water content approaches the liquid limit, clay transitions from a plastic to a liquid consistency. The undrained strength of clay in the liquid state is very low, indicating that changes in state and water content directly affect its strength. This highlights the intrinsic link between undrained shear strength and water content, as defined by the Atterberg limits. In this regard, the fall cone test is one of the most appropriate tests for determining not only these limits but also the undrained shear strength of soil. This paper first examines the fall cone test and the relationship between its penetration and water content, followed by the evaluation of undrained shear strength derived from this method.

According to several authors [4–6], the undrained shear strength at a water content corresponding to the liquid limit represents a characteristic value for all soil types. These studies have shown that the undrained strength of fine-grained soils at the liquid limit falls within a narrow range of approximately 1.3 to 2.8 kPa. WROTH and WOOD [6] were among the first to experimentally confirm that undrained shear strength at the plastic limit is approximately 100 times greater than at the liquid limit, with average values ranging from 1.7 to 170 kPa. Similar findings were reported by SHARMA and BORA [7], confirming the same average values as Wroth and Wood. KARAKAN [8] reported that the average undrained shear strength of clay mixtures at the plastic limit was 166.77 kPa, while at the liquid limit it was 1.67 kPa. In addition, several authors [9–15] have established

empirical correlations that allow estimation of undrained shear strength based on known water content. Understanding the relationship between water content and undrained shear strength is of particular importance in engineering practice, where rapid strength assessment of fine-grained soils is often required during preliminary design stages. Undrained shear strength was determined using the fall cone method with two cones: 30°/80 g and 60°/60 g. In this context, the present study focuses exclusively on remoulded soil samples, enabling controlled assessment of the relationship between water content and undrained shear strength without the influence of natural soil structure.

Materials and methods. The soil samples selected for this analysis are part of a project involving the construction of a bypass around Belgrade and Pančevo, stretching from Bujanj Potok to the state road I class Pančevo-Vršac, located on the left bank of the Danube, where Quaternary sediments – both alluvial and aeolian are present. Geomechanical laboratory tests were conducted on eight soil samples. According to the USCS classification, they belong to silty clays of medium plasticity (CL), with natural moisture content ranging from 20% to 34%, plastic limit values between 19% and 23%, liquid limit values between 41% and 50% and unit weight between 18.1 and 20.4 kN/m³. A summary of the basic identification parameters for the soil samples is shown in Table 1. Each sample was tested across a wide range of water contents, resulting in more than 180 individual measurements of cone penetration and undrained shear strength. The dataset therefore represents repeated measurements on the same material rather than a limited set of observations. The analysis focuses on identifying within-sample trends through regression models and on normalized relationships.

T a b l e 1
Soil identification and classification characteristics

Sample ID (m)	γ	γ_d	w	G_s	e	Sr	w_L	w_P	Ip	Ic
	kN/m ³		%	–	–	%	%	%	%	%
PB-27 (1.1–1.5)	20.2	16.7	20	2.58	0.512	100	41	19	22	0.956
MB-25 (1.3–1.6)	19.8	16.2	23.1	2.57	0.563	100	49	23	26	0.996
MB-13 (1.5–1.8)	18.1	13.8	34	2.54	0.805	100	42	24	18	0.450
D-1 (2.0–2.2)	19.7	16.0	26.3	2.61	0.624	100	40	22	18	0.761
ABo-23 (2.5–2.8)	19.7	15.8	24.7	2.63	0.632	100	44	24	20	0.965
PB-27 (4.3–4.6)	19.6	16.1	22.2	2.57	0.567	100	42	21	21	0.945
ABo-41 (7.6–7.9)	20.1	16.3	24.3	2.59	0.558	100	47	22	25	0.907
ABo-41 (12.7–13)	20.4	16.7	24.6	2.62	0.538	100	50	22	28	0.908

The fall cone method, originally developed by the Geotechnical Commission of the Swedish State Railways and described by HANSBO [4], was used to determine the undrained shear strength of the investigated soils. The method is based on measuring the penetration depth of a cone with a defined mass and apex angle

into a prepared soil specimen. In this study, the fall cone test was performed to determine the liquid limit (w_L) and to evaluate the undrained shear strength using two cone configurations: 30°/80 g and 60°/60 g. Undrained shear strength is expressed using the following equation according to the SRPS EN ISO 17892-12:2017 [16] standard:

$$(1) \quad c_u = \frac{c \cdot Q}{h^2},$$

where c_u is the undrained shear strength of the soil sample, c is a constant that depends on the type of cone (its angle, mass, and the applied standard), Q is the weight of the cone, and h is the cone penetration depth.

Results and discussion. The undrained shear strength c_u depends on the penetration depth of the cone h , and thus indirectly on the moisture content of the sample. The wetter the soil, the deeper the cone penetrates, resulting in lower undrained shear strength. This dependence can be approximated by a logarithmic or exponential relationship, depending on the mineralogical composition and plastic characteristics of the sample. The liquid limit w_L is determined by the fall cone method, while the plastic limit w_P is determined by the rolling method. If the plastic limit is to be obtained by the fall cone method, the undrained shear strength characteristic for that soil moisture must be known. Many researchers have considered that it is 100 times greater than the undrained shear strength at the liquid limit, and therefore it is denoted as w_{P100} [7, 8, 17].

In these studies, the undrained shear strength was calculated according to the SRPS EN ISO 17892-12:2017 standard [16], where the undrained shear strength values of 1.57 and 1.59 kPa were obtained for cones of types 30°/80 g and 60°/60 g, respectively. To equalize the undrained shear strength values at the liquid limit for both cone types, a coefficient value of 0.81 was used for the 30°/80 g cone. Thus, the undrained shear strength at the liquid limit for both cone types is 1.59 kPa. The adopted values for undrained shear strength are 1.6 kPa at the liquid limit and 160 kPa at the plastic limit. Figure 1A shows the relationship between moisture content (w) and undrained shear strength (c_u) for a total of 95 soil samples tested with a 30°/80 g cone and 88 samples tested with a 60°/60 g cone, together with literature data.

Specifically, for the investigated samples, the linear relationship between $\log(w) - \log(c_u)$ has the form:

For cone 30°/80 g:

$$(2) \quad \log c_u = \log(8 \cdot 10^{10}) - 6.442 \cdot \log w$$

For cone 60°/60 g:

$$(3) \quad \log c_u = \log(5 \cdot 10^{10}) - 6.326 \cdot \log w.$$

Eqs (2) and (3) are scaled and normalized to the same basis for comparison purposes. The undrained shear strength c_u is divided by the adopted undrained shear strength at the liquid limit $c_{uLL} = 1.6$ kPa, and the moisture content is divided by the liquid limit w_L . The grouped data are shown in Fig. 1B. For the tested samples, the undrained shear strength can be calculated based on the known moisture content and liquid limit as follows:

For cone 30°/80 g:

$$(4) \quad c_u = 1.6 \cdot WCR^{-6.653}$$

For cone 60°/60 g:

$$(5) \quad c_u = 1.6 \cdot WCR^{-6.873},$$

where WCR is the ratio of the moisture content to the liquid limit.

By knowing the liquid limit of the soil and the corresponding water content, the undrained shear strength was calculated using Eqs (4) and (5). Along with the measured values of undrained shear strength obtained by the fall cone method (for both cone types), the results are presented in the diagram (Fig. 1C).

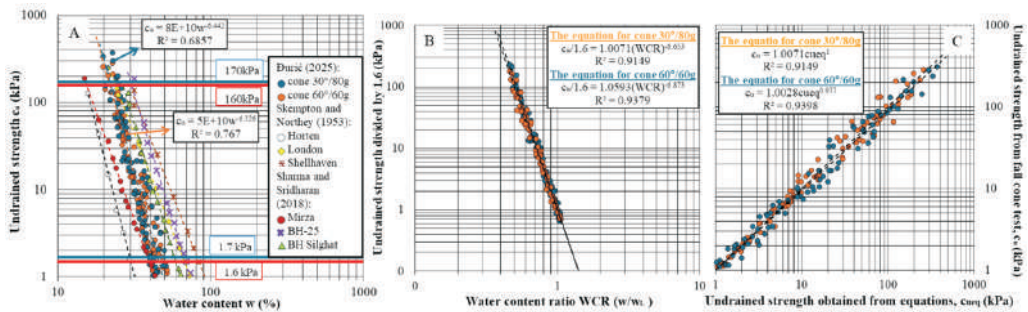


Fig. 1. A) Logarithmic relationship between undrained shear strength and water content for both cone types; B) Relationship between divided undrained shear strength and water content ratio; C) Relationship between measured undrained shear strength and obtained from equations

Statistical analyses confirm that the undrained shear strength of the soil can be calculated using Eqs (4) and (5). This conclusion is supported by the agreement between laboratory and calculated results for both cones (30°/80 g and 60°/60 g), as presented in Fig. 1C.

To evaluate the predictive accuracy of the proposed empirical model for both cones, statistical error metrics were calculated. For the 30°/80 g cone, the Root Mean Square Error (RMSE) was 16.8 kPa, while the Mean Absolute Error (MAE) was 8.04 kPa. For the 60°/60 g cone, the RMSE was 20.6 kPa and the MAE was 9.3 kPa. These values indicate a moderate level of agreement between predicted and measured undrained shear strength values. Although the model performs reasonably well across the full range of samples, larger deviations were observed

for softer soils with lower strength values. These deviations at the individual measurement level increase the overall RMSE and MAE values. Nevertheless, the predictive performance of the model is considered adequate for preliminary engineering evaluations, where rapid estimates of undrained shear strength are required. For detailed geotechnical design, however, additional calibration and validation using a broader dataset are recommended. Although the overall relationship between undrained shear strength and water content ratio is evident, the influence of repeated measurements within individual samples requires additional examination.

Within-sample regression analysis of undrained shear strength. Although the number of investigated soil samples is limited, each sample was tested over a wide range of water contents, resulting in many repeated measurements. To properly account for this data structure, a within-sample regression analysis was performed. For each soil sample, the relationship between undrained shear strength and water content ratio was examined using a log-log regression of the form:

$$(6) \quad \log(c_u) = a_i + b_i \cdot \log(WCR),$$

where a_i and b_i are sample-specific regression coefficients. The results of the regressions, together with the number of tests and coefficients of determination, are summarized in Table 2.

T a b l e 2

The results of within-sample regression analysis

Cone tip:	Cone 30°/80 g					Cone 60°/60 g				
Sample ID (m)	Number of tests	a	SE(a)	b	SE(b)	Number of tests	a	SE(a)	b	SE(b)
PB-27 (1.1–1.5)	13	0.22	0.02	−7.19	0.12	12	0.18	0.04	−7.76	0.21
MB-25 (1.3–1.6)	15	0.22	0.04	−6.51	0.18	14	0.32	0.05	−6.46	0.22
MB-13 (1.5–1.8)	10	0.20	0.03	−6.36	0.30	9	0.16	0.02	−6.29	0.23
D-1 (2.0–2.2)	9	0.20	0.02	−7.67	0.13	13	0.25	0.02	−7.40	0.18
ABO-23 (2.5–2.8)	14	0.23	0.03	−6.54	0.20	13	0.16	0.03	−7.03	0.24
PB-27 (4.3–4.6)	13	0.13	0.03	−6.87	0.18	9	0.23	0.03	−6.33	0.19
ABO-41 (7.6–7.9)	9	0.20	0.01	−6.42	0.07	10	0.28	0.03	−6.71	0.16
ABO-41 (12.7–13)	12	0.24	0.03	−5.94	0.16	8	0.39	0.02	−6.32	0.12
Mean value:	–	0.20	0.03	−6.69	0.17	–	0.25	0.03	−6.79	0.19

The obtained results indicate consistently high coefficients of determination for all samples, with R^2 values ranging from 0.96 to 1.00 for the 30°/80 g cone and from 0.98 to 1.00 for the 60°/60 g cone, confirming a strong dependence between undrained shear strength and water content ratio. In addition, all regression coefficients were statistically significant ($p < 0.001$), indicating that the estimated

effects are statistically significant. To further investigate the origin of the observed differences in regression intercepts between individual samples, unit weight was introduced as an additional explanatory variable in a multiple regression framework. Using all available measurements, the $\log(c_u)$ was expressed as a function of the $\log(WCR)$ and unit weight (γ):

$$(7) \quad \log(c_u) = a_0 + a_1 \cdot \log(WCR) + a_2 \cdot \gamma.$$

The regression coefficients obtained for the two cone configurations are summarized by their estimated values, standard errors (SE), and p -values. For the $30^\circ/80$ g cone, the corresponding coefficients were -0.02 ($SE = 0.30$, $p = 0.96$) for the intercept, -6.64 ($SE = 0.09$, $p < 0.001$) for the water content ratio, and 0.01 ($SE = 0.02$, $p = 0.46$) for unit weight. For the $60^\circ/60$ g cone, the intercept was -1.61 ($SE = 0.32$, $p < 0.001$), the coefficient for the water content ratio was -6.75 ($SE = 0.10$, $p < 0.001$), and the coefficient for unit weight was 0.09 ($SE = 0.02$, $p < 0.001$). Both models achieved a high coefficient of determination ($R^2 = 0.98$). The estimated coefficients for the water content ratio were highly consistent between the two cone configurations and statistically significant, confirming that moisture variation is the dominant factor governing the reduction in undrained shear strength. In contrast, the effect of unit weight was positive in both models but statistically significant only for the $60^\circ/60$ g cone, indicating that its contribution is comparatively smaller and more dependent on cone configuration. Overall, the results indicate that the water content ratio is the primary and most robust predictor of undrained shear strength, whereas the influence of unit weight is secondary and configuration dependent.

From a mixed-effects modelling perspective, the effect of the water content ratio may be interpreted as a global (fixed) effect common to all samples, while differences between regression intercepts reflect sample-specific characteristics. In this context, unit weight can be regarded as a partial explanatory variable for these intercept differences, whereas the remaining unexplained variability may be attributed to random sample effects related to mineral composition and stress history. Although a full mixed-effects model was not explicitly fitted, this conceptual framework provides a statistically consistent interpretation of the observed regression behaviour and enables effective use of all available experimental data.

Verification of results using literature data and python programming language. For this analysis, the Python programming language was used, along with the libraries pandas, numpy, and matplotlib for data processing and visualization. The undrained shear strength of soil (c_u) was calculated mathematically using eight empirical expressions (Fig. 2), depending on the variation of the coefficient WCR (water content ratio). The obtained results are presented in a combined diagram (Fig. 2) to enable comparison between the expressions and a visual assessment of the relationships among the parameters used. The axes were

adjusted so that the diagram starts from physically relevant values ($WCR \geq 0.4$, $c_u \geq 0$).

Known empirical models from the literature were compared with the model derived from experimental data obtained through this study and presented in Fig. 2.

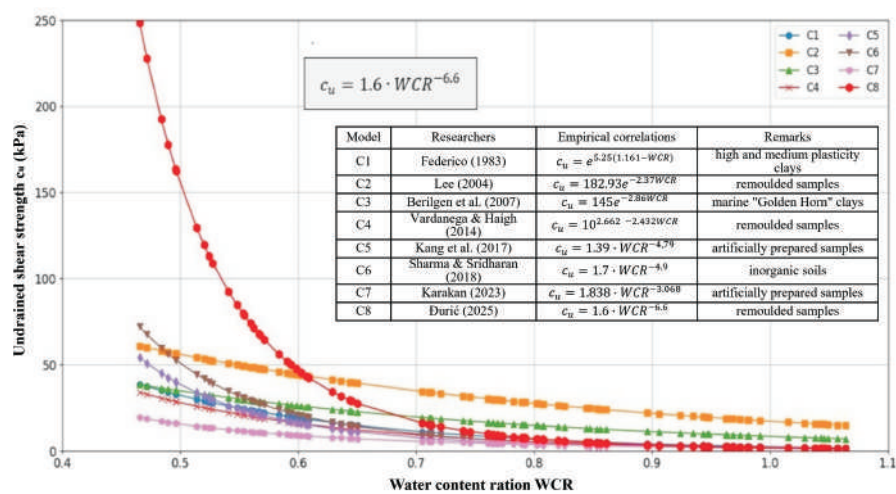


Fig. 2. Graphical representation of various empirical models

When observing all the curves on a single diagram, it can be noted that the models show very good agreement at high moisture ratios and when undrained shear strengths are low. For soils with higher undrained shear strengths ($c_u > 25$ kPa), a specific empirical expression must be defined for each material. Most models have been developed for marine clays or very soft soils with low strength and high water content, which is not the case with the materials investigated in this study. Therefore, each model should be applied in practice with caution, following a thorough preliminary analysis and with a solid understanding of the basic phase properties of the soil.

Conclusion. This study examined the relationship between water content, fall cone penetration, and undrained shear strength of remoulded medium-plasticity silty clay. The results confirm that the fall cone test provides a reliable basis for assessing undrained shear strength, with a clear dependence on water content described by power-law relationships. Within-sample regression analyses revealed consistently high coefficients of determination, indicating that undrained shear strength reduction with increasing water content is largely independent of specific soil samples. The use of the water content ratio enabled comparison with existing empirical models, showing good agreement at higher moisture contents while highlighting the need for local calibration at higher strength levels. Overall, the findings reinforce the applicability of fall cone testing as an effective method

for rapid strength assessment in geotechnical practice. The proposed relationship may be used for rapid preliminary estimation of undrained shear strength for the investigated remoulded soil, within the investigated range of water content ratios ($WCR \approx 0.4-1.1$), during preliminary site investigations and early design stages. Nevertheless, extrapolation beyond the tested soil type and WCR range should be avoided without additional validation against independent datasets.

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