

A NEW APPROACH TO DETERMINING ACTIVE POWER LOSSES IN THE ELECTRIC POWER SYSTEM

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Abstract

The article proposes a method through which a differentiated consideration of active power losses in electric power systems (EPS) is formulated. Its applicability and efficiency in the study and operation of real electric power systems are substantiated.

Key words: active power losses, power factor, unbalanced and nonsinusoidal current

Introduction. The IEEE standard [1, 2] develops a theoretical framework for the analysis of a “Three-phase nonsinusoidal and unbalanced system”, introducing fundamental components of voltage and current associated with the first harmonic, as well as non-fundamental components associated with higher harmonics of voltage and current. In the theoretical framework adopted in IEEE Std 1459–2010 [2], various researchers have identified [3, 4] significant discrepancies in the accuracy of the formulas for determining currents and voltages used in the calculation of different types of power. In order to eliminate these shortcomings, [5, 6] propose that the effective value of the apparent power be calculated without including the neutral current in the value of the total effective current. The inaccurate computational procedures, as well as the complex definitions of

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the different power quantities, make this method unconvincing and inapplicable for practical use [7–10].

Partial approach to determining active power losses. In four-wire networks with a grounded star point, comprising generating sources and transfer facilities, it is acceptable in practical studies to assume that the voltage system of the three phases and the neutral, U_A , U_B , U_C and U_o , is symmetrical, balanced and sinusoidal, since in most of the cases considered, the requirements of the various standards for voltage quality are satisfied. Under this condition, and in the presence of an unbalanced and nonsinusoidal load, the instantaneous values of the powers for the three phases, after applying the Fourier series and the Fortescue symmetrical components method, are represented by the following expressions [11, 12]:

$$\begin{aligned}
 p_A &= \frac{1}{3} R_e \left(\dot{S} + \dot{S}^* \cdot e^{j2\omega t} \right) + \frac{1}{3} R_e \left(\dot{N} + \dot{N}^* \cdot e^{j2\omega t} \right) \\
 &\quad + \frac{1}{3} R_e \left(\dot{G} + \dot{G}^* \cdot e^{j2\omega t} \right) + u_A \cdot \sum_{y=2}^{\infty} i_{Ay}, \\
 p_B &= \frac{1}{3} R_e \left(\dot{S} + a \cdot \dot{S}^* \cdot e^{j2\omega t} \right) + \frac{1}{3} R_e \left(a \cdot \dot{N} + \dot{N}^* \cdot e^{j2\omega t} \right) \\
 (1) \quad &\quad + \frac{1}{3} R_e \left(a^2 \cdot \dot{G} + \dot{G}^* \cdot e^{j2\omega t} \right) + u_B \cdot \sum_{y=2}^{\infty} i_{By}, \\
 p_C &= \frac{1}{3} R_e \left(\dot{S} + a^2 \cdot \dot{S}^* \cdot e^{j2\omega t} \right) + \frac{1}{3} R_e \left(a^2 \cdot \dot{N} + \dot{N}^* \cdot e^{j2\omega t} \right) \\
 &\quad + \frac{1}{3} R_e \left(a \cdot \dot{G} + \dot{G}^* \cdot e^{j2\omega t} \right) + u_C \cdot \sum_{y=2}^{\infty} i_{Cy},
 \end{aligned}$$

where $\dot{S}$ and $\dot{S}^*$ are the complex and complex-conjugate values of the apparent power S , $\dot{N}$ and $\dot{N}^*$ are the complex and complex-conjugate values of the pulsating power N , $\dot{G}$ and $\dot{G}^*$ are the complex and complex-conjugate values of the latent power G , a is the Steinmetz operator [11]

$$a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, \quad 1 + a + a^2 = 0, \quad a^3 = 1, \quad a^4 = a,$$

and ω is the angular frequency of the rotating vectors S , N , and G .

System of equations (1) includes four types of powers, referred to as partial powers (S_n , N_n , G_n , and D_n), since they relate to fundamentally different power characteristics [11, 13].

• **First group of partial powers (S_n):**

$$\begin{aligned}
 S_{\Pi A} &= \frac{1}{3} R_e \left(\dot{S} + \dot{S}^* \cdot e^{j2\omega t} \right), \\
 S_{\Pi B} &= \frac{1}{3} R_e \left(\dot{S} + a \cdot \dot{S}^* \cdot e^{j2\omega t} \right), \\
 S_{\Pi C} &= \frac{1}{3} R_e \left(\dot{S} + a^2 \cdot \dot{S}^* \cdot e^{j2\omega t} \right).
 \end{aligned}
 \tag{2}$$

Expressions (2) have two components – a constant component, independent of time t , namely $\frac{1}{3} R_e (\dot{S})$, and variable components dependent on t , respectively, $\frac{1}{3} R_e \cdot \dot{S}^* \cdot e^{j2\omega t}$, $\frac{1}{3} R_e \cdot a \cdot \dot{S}^* \cdot e^{j2\omega t}$, and $\frac{1}{3} R_e \cdot a^2 \cdot \dot{S}^* \cdot e^{j2\omega t}$. The sum of the instantaneous values of the partial powers from (2) is represented by the expression:

$$S_{\Pi} = S_{\Pi A} + S_{\Pi B} + S_{\Pi C} = \frac{1}{3} R_e \left[3 \cdot \dot{S} + (1 + a + a^2) \cdot \dot{S}^* \cdot e^{j2\omega t} \right] = R_e \left[\dot{S} \right] = P.
 \tag{3}$$

Therefore, the sum of the instantaneous values of the partial powers represents the active power P of the entire system, which is distorted with respect to the currents. In an analogous manner, the following can be written:

$$Q = I_m \left[\dot{S} \right], \quad S = \sqrt{P^2 + Q^2},
 \tag{4}$$

where Q is the reactive power of the distorted system.

The average value of the sum (4) within one period T is determined by means of the expression:

$$\frac{1}{T} \int_0^T S_{\Pi} \cdot dt = \frac{1}{T} \int_0^T (S_{\Pi A} + S_{\Pi B} + S_{\Pi C}) dt = R_e \left[\dot{S} \right] = P.
 \tag{5}$$

• **Second group of partial powers (N_{Π}):**

$$\begin{aligned}
 N_{\Pi A} &= \frac{1}{3} R_e \left(\dot{N} + \dot{N}^* \cdot e^{j2\omega t} \right), \\
 N_{\Pi B} &= \frac{1}{3} R_e \left(a \cdot \dot{N} + \dot{N}^* \cdot e^{j\omega t} \right), \\
 N_{\Pi C} &= \frac{1}{3} R_e \left(a^2 \cdot \dot{N} + \dot{N}^* \cdot e^{j\omega t} \right).
 \end{aligned}
 \tag{6}$$

Here again, the partial powers of the individual phases have constant components, respectively, $\frac{1}{3} R_e \left[\dot{N} \right]$, $\frac{1}{3} R_e \left[a \cdot \dot{N} \right]$ and $\frac{1}{3} R_e \left[a^2 \cdot \dot{N} \right]$, and a variable component $\frac{1}{3} R_e \left[\dot{N}^* \cdot e^{j2\omega t} \right]$. The sum of the instantaneous values of these partial powers is:

$$N_{\Pi} = N_{\Pi A} + N_{\Pi B} + N_{\Pi C} = \frac{1}{3} R_e \left[(1 + a + a^2) \dot{N} + \dot{N}^* \cdot e^{j2\omega t} \right] = R_e \left[\dot{N}^* \cdot e^{j2\omega t} \right].
 \tag{7}$$

It follows from (7) that N_{Π} has only a variable component, as the vector $\dot{N}$ rotates with double angular frequency 2ω and with amplitude N . The average value of the sum N_{Π} is also zero, as is the case with the reactive power Q

$$(8) \quad \frac{1}{T} \int_0^T N_{\Pi} \cdot dt = \frac{1}{T} \int_0^T (N_{\Pi A} + N_{\Pi B} + N_{\Pi C}) dt = 0.$$

Therefore, in an unbalanced three-phase system with a grounded neutral, during the transfer of active power, the so-called pulsating power N arises. As in the case of reactive power Q , the average value of this power over one period is equal to zero.

• **Third group of partial powers (G_{Π}):**

$$(9) \quad \begin{aligned} G_{\Pi A} &= \frac{1}{3} R_e \left(\dot{G} + \dot{G}^* \cdot e^{j2\omega t} \right), \\ G_{\Pi B} &= \frac{1}{3} R_e \left(a^2 \cdot \dot{G} + a^2 \dot{G}^* \cdot e^{j2\omega t} \right), \\ G_{\Pi C} &= \frac{1}{3} R_e \left(a \cdot \dot{G} + a \cdot \dot{G}^* \cdot e^{j2\omega t} \right). \end{aligned}$$

These partial powers are called “hidden” powers; they have constant components $\frac{1}{3} R_e [\dot{G}]$, $\frac{1}{3} R_e [a \dot{G}]$ and $\frac{1}{3} R_e [a^2 \dot{G}]$, and variable components $\frac{1}{3} R_e [\dot{G}^* \cdot e^{j2\omega t}]$, $\frac{1}{3} R_e [a \cdot \dot{G}^* \cdot e^{j2\omega t}]$ and $\frac{1}{3} R_e [a^2 \dot{G}^* \cdot e^{j2\omega t}]$. The sum of the instantaneous values of the partial hidden powers is:

$$(10) \quad G_{\Pi} = G_{\Pi A} + G_{\Pi B} + G_{\Pi C} = \frac{1}{3} R_e \left[(1 + a + a^2) \dot{G} + (1 + a + a^2) \cdot \dot{G}^* \cdot e^{j2\omega t} \right] = 0$$

Therefore, the average value of this partial power is also zero

$$(11) \quad \frac{1}{T} \int_0^T G dt = \frac{1}{T} \int_0^T (G_{\Pi A} + G_{\Pi B} + G_{\Pi C}) dt = 0.$$

The above makes it possible to conclude that in an unbalanced four-wire system with a grounded neutral, during the transfer of active power P , hidden power G also arises, the average value of which over one period is zero.

• **Fourth group of partial powers (D_{Π}):**

$$(12) \quad \begin{aligned} D_{\Pi A} &= u_A \sum_{v=2}^{\infty} i_{Av} \\ D_{\Pi B} &= u_B \sum_{v=2}^{\infty} i_{Bv} \\ D_{\Pi C} &= u_C \sum_{v=2}^{\infty} i_{Cv}. \end{aligned}$$

These partial powers constitute the so-called distortion power D , which is determined by the level of the higher current harmonics in the individual phases (i_{Av} , i_{Bv} , i_{Cv}). The average value of D within one period and the average value of the sum of the instantaneous values of the power of the three phases are represented as products of current and voltage harmonics with different numbers, and these values are zero

$$(13) \quad \frac{1}{T} \int_0^T D_{\Pi} \cdot dt = \frac{1}{T} \int_0^T (D_{\Pi A} + D_{\Pi B} + D_{\Pi C}) dt = 0.$$

Therefore, under nonsinusoidal load currents, during the transfer of active power P , distortion power D arises, the average value of which over one period is zero. The presented analysis makes it possible to draw the following important conclusion:

The four indicated types of power (Q , N , G , and D), which arise during the transfer of active power P from the source to the consumers, do not participate directly in this transfer, since their average value within one period is equal to zero. However, they are part of the total current that determines the total active power losses, i.e. these power quantities have a significant share in the total active power losses and play an essential role in the electric power efficiency of the EPS and all its subdivisions [11].

In accordance with the presented partial approach, the so-called actual apparent power $S_{\mathcal{A}}$ is defined, which can be expressed by means of the following equation [4, 11, 14]:

$$(14) \quad S_{\mathcal{A}} = \left(3U^2 \left[3(I_1)_1^2 + 3(I_2)_1^2 + 3(3n+1)(I_0)_1^2 \right] + 3U^2 \left(\sum_{v=2}^{\infty} I_{av}^2 + \sum_{v=2}^{\infty} I_{bv}^2 + \sum_{v=2}^{\infty} I_{cv}^2 + n \sum_{v=2}^{\infty} I_{Nv}^2 \right) \right)^{1/2},$$

where $(I_1)_1$, $(I_2)_1$, and $(I_0)_1$ are the positive-, negative-, and zero-sequence currents of the fundamental harmonic, I_1 , I_{Av} , I_{Bv} , I_{Cv} , and I_{Nv} are the v -th harmonics of the currents of phases A, B, and C, and of the neutral N , respectively, U is the phase voltage ($U_A = U_B = U_C = U$).

In fact, the first term of (14) can be represented as follows:

$$(15) \quad S_{\mathcal{A}} = \sqrt{P^2 + Q^2 + N^2 + (3n+1) \cdot G^2 + D^2}.$$

The power factor PF represents the ratio of the active power P to the actual apparent power $S_{\mathcal{A}}$. In accordance with the partial power technology and

expression 15, PF is written as follows [11]:

$$(16) \quad PF = \frac{P}{S_{\text{д}}} = \frac{P}{\sqrt{S^2 + N^2 + (3n+1) \cdot G^2 + D^2}} = \frac{P}{\sqrt{P^2 + Q^2 + N^2 + (3n+1) \cdot G^2 + D^2}}.$$

For asymmetric, unbalanced and nonsinusoidal currents, the active power losses ΔP in the system elements and during the transfer of active power P are determined by the following expression:

$$(17) \quad \Delta P \approx \frac{1}{PF} = \frac{1}{P^2} \cdot [P^2 + Q^2 + N^2 + (3n+1) \cdot G^2 + D^2].$$

In accordance with (17), the weighted participation, expressed as a percentage of all power components in the total active power losses ΔP can be determined, represented as a partial share:

- Partial share of reactive power losses $\Delta P_Q = \frac{Q^2}{P^2} \cdot 100 [\%];$
- Partial share of pulsating power losses $\Delta P_N = \frac{N^2}{P^2} \cdot 100 [\%];$
- Partial share of hidden power losses $\Delta P_G = \frac{G}{P^2} \cdot 100 [\%];$
- Partial share of distortion power losses $\Delta P_D = \frac{D^2}{P^2} \cdot 100 [\%].$

Conclusion.

1. An effective and practically applicable approach for determining different types of power is presented. In accordance with (15), and proceeding from the thermal nature of power and electric energy losses, the actual apparent power $S_{\text{д}}$ is defined as an equivalent active power P , which is dissipated in a symmetrical, balanced and undistorted system while producing the same active power losses over the same resistances. It also follows from (15) that the process of transferring active power P in any distorted EPS is accompanied by the generation of four power quantities: reactive power Q ; pulsating power N , caused by current asymmetry, i.e. negative-sequence components; latent power G , caused by current unbalance, i.e. zero-sequence components; and distortion power D , caused by the presence of higher current harmonics. The power factor PF is a power characteristic that fully reflects the energy-related nature of the processes occurring under asymmetric, unbalanced and nonsinusoidal current regimes in the EPS.

2. The active power losses caused by the partial quantities of the four types of power adequately and comprehensively characterize the specific features of the energy processes in the EPS.

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